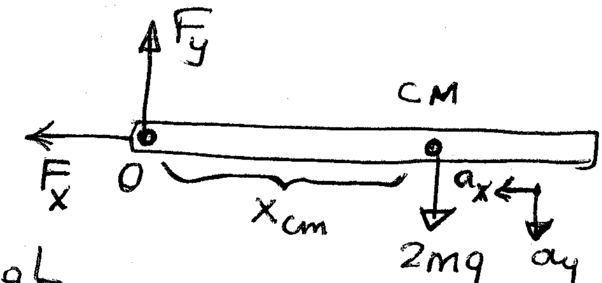
KEY QUESTION 1
(No partial credit unless (a) and (b) are correct)

a) $I_0 = \frac{1}{3} mL^2 + mL^2 = \frac{4}{3} mL^2$

b) $x_{cm} = \frac{mL/2 + mL}{m+m} = \frac{3}{4} L$

$y_{cm} = 0$

c) Free Body Diagram



$\tau_0 = I_0 \cdot \alpha$

$\tau_0 = x_{cm} \cdot 2mg = \frac{3}{2} mgL$

$\Rightarrow \frac{3}{2} mgL = \frac{4}{3} mL^2 \cdot \alpha \Rightarrow \alpha = \frac{9}{8} \cdot \frac{g}{L}$

d) We first find angular velocity ω by using the conservation of energy

$U_{initial} = K_{final} \Rightarrow 2mg \cdot \frac{3L}{4} = \frac{1}{2} I_0 \omega^2$

$\Rightarrow 2mg \cdot \frac{3L}{4} = \frac{1}{2} \cdot \frac{4}{3} mL^2 \omega^2 \Rightarrow \omega^2 = \frac{9}{4} \cdot \frac{g}{L}$

$a_x = a_{centripetal} = \omega^2 \cdot \frac{3L}{4} = \frac{27g}{16}$ (in $-x$ direction)

$a_y = a_{tangential} = \alpha \cdot \frac{3L}{4} = \frac{27g}{32}$ (in $-y$ direction)

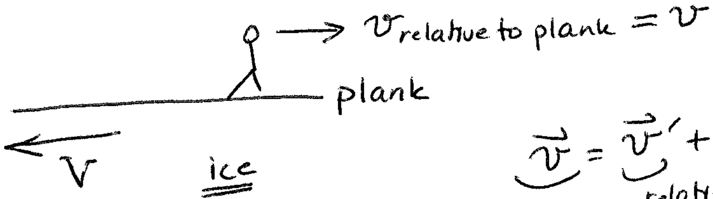
e) $F_x = 2m \cdot a_x = \frac{27}{8} mg$ (in $-x$ direction)

$2mg - F_y = 2m a_y$

$\Rightarrow F_y = 2mg - 2ma_y = 2m \left[g - \frac{27g}{32} \right]$

$\Rightarrow F_y = \frac{5}{16} mg$

Q4



$$\underbrace{\vec{v}}_{\substack{\text{rel to}\\ \text{ice}\\ \text{surface}}} = \underbrace{\vec{v}\,'}_{\substack{\text{relative}\\ \text{to plank}}} + \underbrace{\vec{V}}_{\text{plank}}$$

$$v_{ice} = v - V$$

mom. cons: $0 = -3mV + m(v - V)$

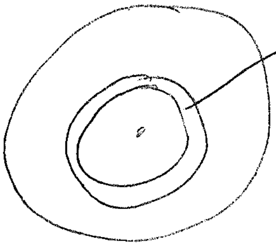
a) $V = v/4$) ~~boat~~ plank $\leftarrow$

$$\vec{v} = \vec{v}\,' + \vec{V}$$

b) $v_{ice} = v - v/4$
$= 3v/4$ //

Very common error: a) $= v/3 \leftarrow$ b) $= 2v/3 \rightarrow$) will get 2 pts for part b: $v_{ice} = v - v/3 = 2v/3$

Q1



$da = 2\pi r\,dr$

$dm = \sigma\,da$
$= a 2\pi r^2 dr$

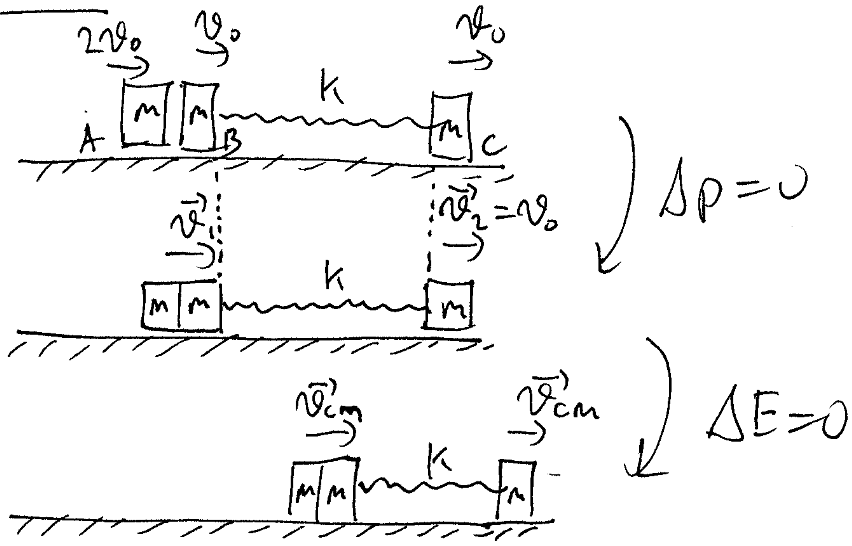
$$M = \int_0^R dm = 2\pi a \frac{r^3}{3}\Big|_0^R = 2\pi a \frac{R^3}{3} \quad //$$

$$I = \int r^2 dm = \int_0^R 2\pi a r^4 dr = 2\pi a \frac{R^5}{5} \quad //$$

$$\left.\begin{aligned} I_A &= I_{cm} + MR^2 \\ &= \frac{2\pi a R^5}{5} + \left(2\pi a \frac{R^3}{3}\right)R^2 \end{aligned}\right\}$$ 1 pt if idea is correct

$$= \frac{16\pi a R^5}{15}$$

Answer 3



1pts

a) $\boxed{v_2 = v_0}$ No impact on mass-C

$\Delta p = 0$: $m(2v_0) + m v_0 + \cancel{m v_0} = (2m) v_1 + \cancel{m v_0}$

$$\boxed{v_1 = \frac{3}{2} v_0}$$ 1 pt

b) $I = P_{B,f} - P_{B,i} = m v_1 - m v_0 = m(3/2 - 1) v_0 = \boxed{\frac{1}{2} m v_0}$ 1 pt

c) $v_{cm} = \frac{1}{m+m+m}\left[2v_0 . m + v_0 m + v_0 m\right] = \boxed{\frac{4}{3} v_0}$ 1 pt

$\Delta E = 0 \Rightarrow$ $\cancel{\frac{1}{2}}(2m)v_1^2 + \cancel{\frac{1}{2}} m v_2^2 = \cancel{\frac{1}{2}}(3m)v_{cm}^2 + \cancel{\frac{1}{2}} k x^2$

$$\cancel{2}m \frac{9}{\cancel{4}_2} v_0^2 + m v_0^2 - \cancel{3}m \frac{16}{\cancel{3}\cdot 3} v_0^2 = k x^2$$

$$\boxed{x = v_0 \sqrt{\frac{m}{6k}}}$$ 1 pt