Boğaziçi University – Department of Physics

2nd Midterm – December 17th, 2010 – 60 Minutes

NO QUESTIONS – NO CALCULATORS – SHOW YOUR WORK CLEARLY

Phys No:	Name:

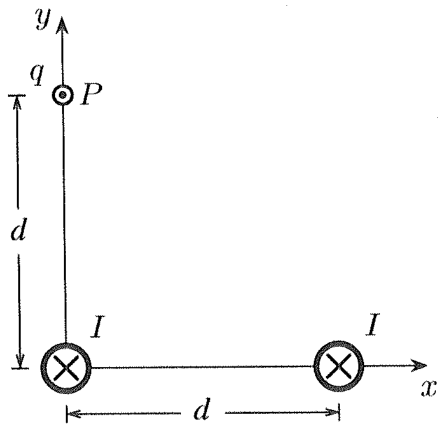
Total

Question 1:

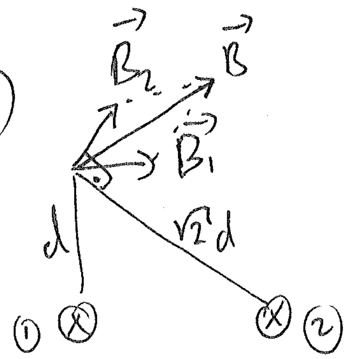
Two long parallel wires carry the same current I directed into the page as shown in the figure.

a) Find the magnetic field vector $\vec{B}$ at the point $P = (0, d, 0)$,

b) Find the force vector on a particle with charge q and velocity $\vec{v} = v_0\hat{k}$ when it is exactly at the point P.



a)



Ampere's Law

$$\vec{B}_1 = \frac{\mu_0 I}{2\pi d}\hat{x}$$

$$\vec{B}_2 = \frac{\mu_0 I}{2\pi(\sqrt{2}d)}\left[\frac{1}{\sqrt{2}}\hat{x} + \frac{1}{\sqrt{2}}\hat{y}\right] = \frac{\mu_0 I}{4\pi d}\left[\hat{x} + \hat{y}\right]$$

$$\vec{B} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 I}{4\pi d}\left[3\hat{x} + \hat{y}\right]$$

b) $$\vec{F} = q\vec{v} \times \vec{B} = q v_0 \frac{\mu_0 I}{4\pi d}\begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & 1 \\ 3 & 1 & 0 \end{pmatrix} = \frac{q v_0 \mu_0 I}{4\pi d}\left[-\hat{x} + 3\hat{y}\right]$$

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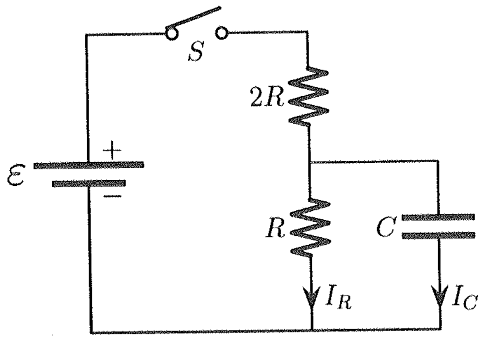
Question 2:

In the circuit shown in figure, the switch S has been closed for a long time. It is then suddenly opened at $t = 0$.

a) Find I_R and I_C before the switch is opened,

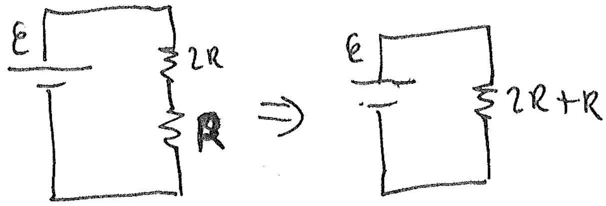
b) Find I_R immediately after the switch is opened,

c) Find $I_R(t)$ after the switch is opened (for $t > 0$).



a) $I_C = 0$ for steady state

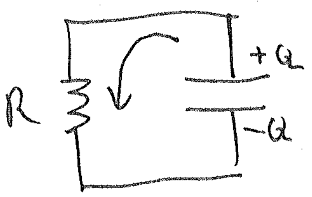
$$I_R = \frac{\varepsilon}{2R+R} = \frac{\varepsilon}{3R}$$



b) Just before the switch is opened, the ΔV_c on capacitor must be same as the ΔV_R on Resistor R,

thus at $t=0$ $\quad \Delta V_c(0) = I_R \cdot R = \frac{\varepsilon}{3R} \cdot R = \frac{\varepsilon}{3}$

When the switch is open, the equivalent circuit is



$\Delta V_c(t=0) = \varepsilon/3$

Thus $I_R = \dfrac{\Delta V_c(t=0)}{R}$

$$\boxed{I_R = \frac{\varepsilon}{3R}} \text{ at } t=0$$

c) Discharging C on R:

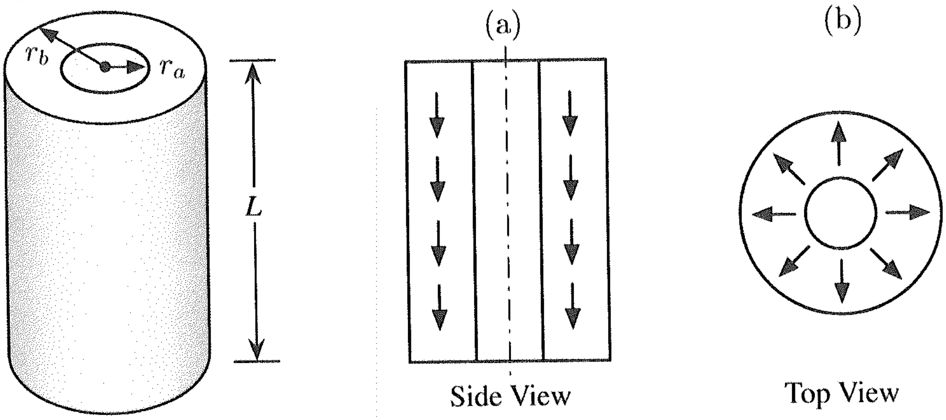
$$I_R(t) = I_R(0) \cdot e^{-t/\tau} = \boxed{\frac{\varepsilon}{3R} e^{-t/RC}}$$

Question 3:

Consider a cylindrical material with resistivity ρ, length L and inner radius of r_a, outer radius of r_b as shown in the figure below. Assume that the current is distributed uniformly over any cross section along the direction of current for each of the following cases. Find the resistance of the system:

a) If the top and bottom surfaces are perfectly conducting terminals of the resistor such that the current flows as shown in figure (a),

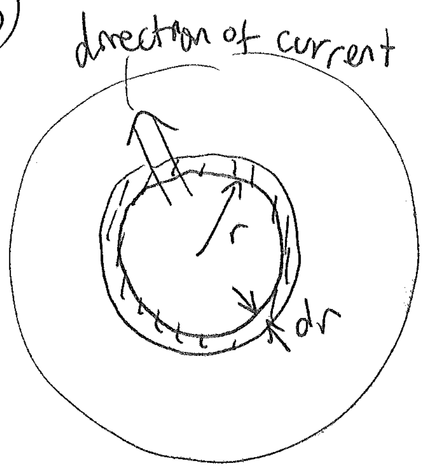
b) If the inner and outer side-surfaces are perfectly conducting terminals of the resistor such that the current flows as shown figure (b).



a) $R = \rho \dfrac{\text{Length}}{\text{cross sectional area}}$ for a constant (uniform) c.s. area along the cylinder.

$$\Rightarrow R = \frac{\rho L}{\pi (r_b^2 - r_a^2)}$$

b)



Consider resistors shaped as cylinderical shells as shown

$$dR = \rho \frac{dr}{2\pi r \cdot L} \Rightarrow \frac{\rho}{2\pi L} \int_{r_a}^{r_b} \frac{dr}{r} = \int dR$$

$$\Rightarrow R = \frac{\rho}{2\pi L} \ln(r_b/r_a)$$

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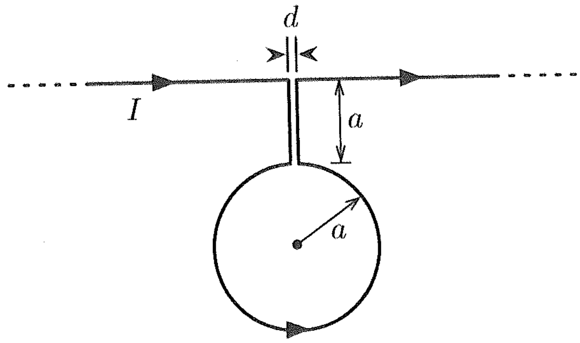
Question 4:

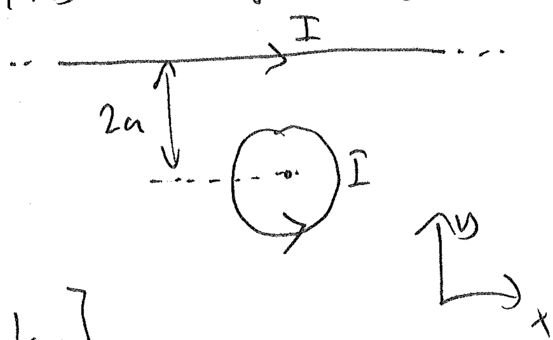
A wire consists of a circular loop of radius a and two infinitely long straight sections connected to the loop by two short segments of length a as shown in the figure. The distance d between the two short segments is negligible ($d \ll a$). The wire lies in the plane of the paper and carries a current I.
Find the direction and magnitude of the magnetic field at the center of the loop.



Small segments

Effects will cancel each other!

Thus two seperate systems



$$\vec{B}_{\text{infinite wire}} = \frac{\mu_0 I}{2\pi(2a)} \underbrace{(-\hat{z})}_{}^{\text{R.H.L.}} \quad [\text{Ampere's Law}]$$

$$\vec{B}_{\text{circle}} = \frac{\mu_0 I}{4\pi} \int_{\text{circle}} \frac{ds}{a^2} (+\hat{z}) = \frac{\mu_0 I}{4\pi a^2} \underbrace{\int_{\text{circle}} ds}_{2\pi a} \hat{z} = \frac{\mu_0 I}{2a} \hat{z}$$

$$\vec{B} = \vec{B}_{\infty \text{ wire}} + \vec{B}_{\text{circle}} = \frac{\mu_0 I}{2a}\left[1 - \frac{1}{2\pi}\right]\hat{z}$$

$$\boxed{\vec{B} = \frac{\mu_0 I (2\pi - 1)}{4\pi a} \hat{z}}$$