Physics Number

Name:

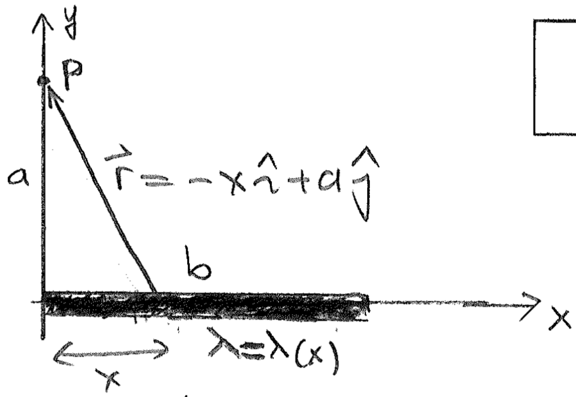
Total Grade:

PHYS 201 FIRST IN-TERM EXAM - Fall '2010

NO QUESTIONS 60 Minutes NO CALCULATORS

S H O W Y O U R W O R K !!

1. A non-uniform line of charge extends along the x-axis, from the origin to $x = b$. Its linear charge density is given by $\lambda = C(x^2 + a^2)^{3/2}$. Find the electric field <u>vector</u> at a point P on the y-axis, at $y = a$ (see figure).



$$\vec{E} = \int \frac{k\,dq}{r^2}\hat{r} = \int \frac{k\,dq}{r^3}\vec{r}$$

$$dq = \lambda\, dx$$

$$\Rightarrow \vec{E} = \frac{1}{4\pi\epsilon_0}\int_0^b \frac{C(x^2+a^2)^{3/2}\,dx\,(-x\hat{i} + a\hat{j})}{(x^2+a^2)^{3/2}}$$

$$= \frac{C}{4\pi\epsilon_0}\left(-\frac{b^2}{2}\hat{i} + ab\,\hat{j}\right)$$

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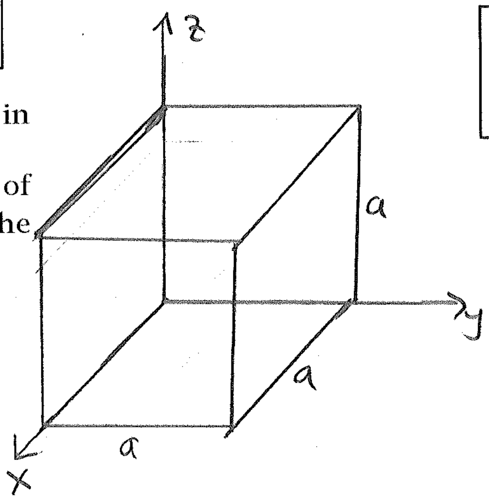
2. An electric field given by $\vec{E} = Cx\hat{i} + Byj$ exists in space.
   (a) Calculate the electric flux coming out of each face of a cube with side a, seen in the figure.
   (b) Calculate the total charge in the cube.



$\Phi = \int \vec{E} \cdot d\vec{A} = \int \vec{E} \cdot \hat{n}\, dA$

a) Back face: $x=0 \Rightarrow \vec{E} = By\hat{j}, \quad \hat{n} = -\hat{i} \Rightarrow \Phi = 0$

Left face: $y=0 \Rightarrow \vec{E} = Cx\hat{i}, \quad \hat{n} = -\hat{j} \Rightarrow \Phi = 0$

Bottom face: $z=0 \Rightarrow \vec{E} = Cx\hat{i} + By\hat{j}, \quad \hat{n} = -\hat{k} \Rightarrow \Phi = 0$

Top face: $z=a \Rightarrow \vec{E} = Cx\hat{i} + By\hat{j}, \quad \hat{n} = \hat{k} \Rightarrow \Phi = 0$

Front face: $x=a \Rightarrow \vec{E} = Ca\hat{i} + By\hat{j}, \quad \hat{n} = \hat{i}$

$\Rightarrow \Phi = \int Ca\, dA = Ca^3$

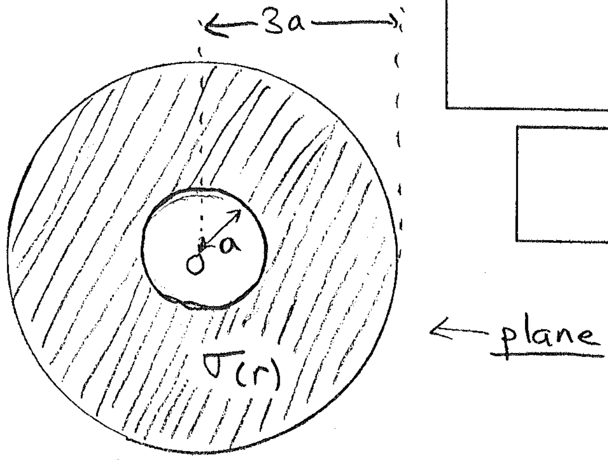
Right face: $y=a \Rightarrow \vec{E} = Cx\hat{i} + Ba\hat{j}, \quad \hat{n} = \hat{j}$

$\Rightarrow$ similarly $\Phi = Ba^3$

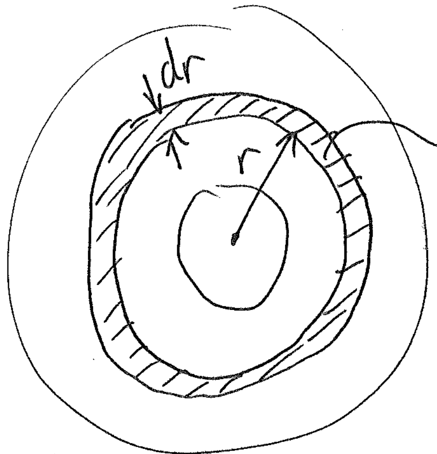
b) Gauss: $\Phi_E = \frac{Q_{in}}{\epsilon_0}$ for closed surfaces

$\Rightarrow \underline{Q_{cube} = \epsilon_0 (B+C) a^3}$

3. Consider the planar object bounded by circles of radii a and $3a$, centered at the origin (see figure). The object is non-uniformly charged, with surface charge density $\sigma = C\,(1\text{-}2a/r)$.
(a) Calculate the total charge of the object.
(b) Calculate the potential at the origin.



a)



$$dA = 2\pi r\,dr$$
$$dq = dA \cdot \sigma$$
$$= 2\pi r \cdot dr \cdot C(1-2a/r)$$

$$Q = \int dq = \int_a^{3a} 2\pi r C(1-2a/r)dr = 2\pi C\int_a^{3a}(r-2a)dr = 2\pi C\left(\frac{r^2}{2}-2ar\right)\Big|_a^{3a}$$
$$= \pi C\,(r^2-4ar)\Big|_a^{3a} = \pi C\left[9a^2-12a^2-a^2+4a^2\right] = \boxed{0}$$

b) $$dV = k\,\frac{dq}{r} = k\,\frac{2\pi \cancel{r}\,dr\,C(1-2a/r)}{\cancel{r}} = 2\pi Ck(1-2a/r)\cdot dr$$

$$V = \int dV = 2\pi Ck\int_a^{3a}(1-2a/r)\cdot dr = 2\pi Ck\,(r-2a\ln r)\Big|_a^{3a}$$
$$= 2\pi Ck\,\big(3a-2a\ln(3a)-a+2a\ln a\big)$$
$$= 2\pi Ck\,(2a-2a\cdot\ln 3)$$
$$\boxed{V = 4\pi Cka(1-\ln 3)} = \boxed{\frac{Ca}{\epsilon_0}(1-\ln 3)}$$

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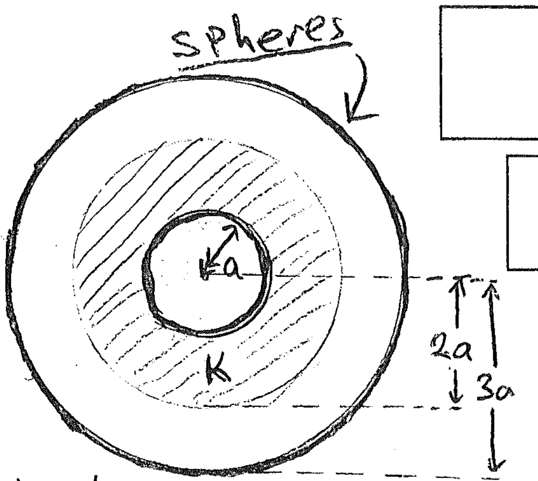
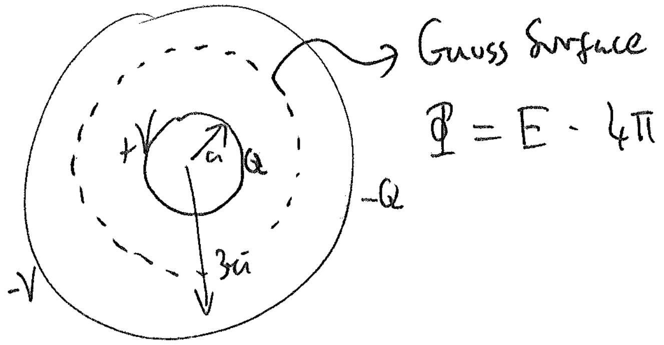
4. A spherical capacitor has inner and outer radii a and $3a$. Part of the space between the plates, namely $a < r < 2a$, is filled with a dielectric described by κ (see figure). Assuming charge $+Q$ on the inner plate, and $-Q$ on the outer,
(a) Find the electric field for all radii.
(b) Calculate the potential difference, hence the capacitance.



a) Let us assume that there is no dielectric at first



$$\Phi = E \cdot 4\pi r^2 = \frac{Q}{\epsilon_0} \Rightarrow \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r} \quad a < r < 3a$$

$$\vec{E} = kQ/r^2 \cdot \hat{r}$$

when we fill it with dielectric material as shown in the question, the electric field will reduce by $1/K$ where there exist dielectric. Thus:

$$\boxed{\vec{E} = \begin{cases} 1/K \cdot kQ/r^2\,\hat{r} & a < r < 2a \\ kQ/r^2\,\hat{r} & 2a < r < 3a \\ \vec{0} & \text{otherwise} \end{cases}}$$

b) $V_- - V_+ = -\int \vec{E} \cdot d\vec{r} \qquad \Delta V_c = V_+ - V_- = \int_{r=a}^{r=3a} \vec{E} \cdot d\vec{r}$

$$\Delta V_c = \frac{kQ}{K}\int_a^{2a}\frac{dr}{r^2} + kQ\int_{2a}^{3a}\frac{dr}{r^2} = \frac{kQ}{K}\left[\frac{-1}{r}\right]_a^{2a} + kQ\left[\frac{-1}{r}\right]_{2a}^{3a}$$

$$= \frac{kQ}{K}\left[\frac{\cancel{2a-a}}{2a\cdot\cancel{a}}\right] + kQ\left[\frac{\cancel{3a-2a}}{3a\cdot 2\cancel{a}}\right] = \frac{kQ}{2aK} + \frac{kQ}{6a} = \frac{kQ}{2a}\left[\frac{1}{K} + \frac{1}{3}\right]$$

$$\boxed{\Delta V_c = \frac{kQ(K+3)}{6Ka}} \Rightarrow C = \frac{Q}{\Delta V_c} \Rightarrow \boxed{C = \frac{6Ka}{k(K+3)}}$$