Summer'2011

Phys No:
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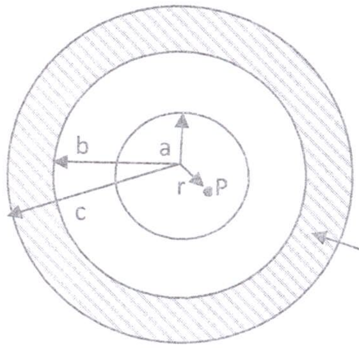
Phys 201
Final

120 min



Q1 (5pt). Consider an insulating sphere with radius a and volumetric charge density $\rho = \frac{2e}{r}$ where e is a constant. The insulating sphere is concentric with a conducting spherical shell with inner radius b, outer radius c and total charge $Q_{TOTAL} = 0$ as shown in the figure. Find the electric field vector $\vec{E}$ at a point P inside the insulating sphere that is r away from the center ($0 < r < a$).

$$Q_{in} = \int_0^r \rho 4\pi r^2 dr$$

$$\Rightarrow Q_{in} = \int_0^r \frac{2e}{r} 4\pi r^2 dr = 4\pi e r^2$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{4\pi e r^2}{\varepsilon_0} \Rightarrow \vec{E} = \frac{e}{\varepsilon_0} \hat{r}$$

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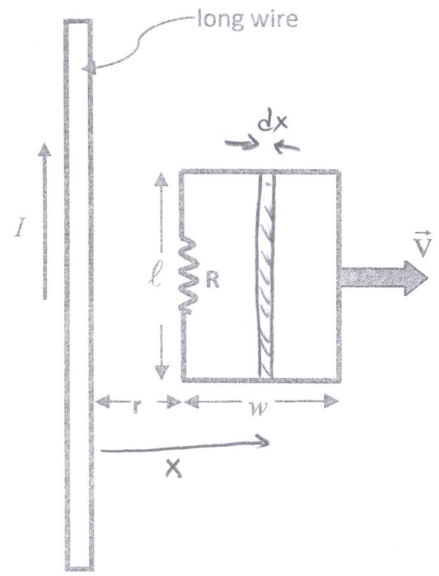
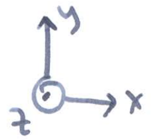


Q2 (5pt). A rectangular loop of dimensions ℓ and w moves with constant velocity $\vec{v}$ away from a long wire that carries a current I in the plane of loop. The total resistance of the loop is R.

(a) Find the magnetic field vector on the plane of the loop using Ampere's law.

(b) Find the magnitude and direction of the induced current in the loop at the instant the near side is at a distance r from the wire.



$\Rightarrow$ direction of B is $(-\hat{k})$ on the plane of the loop.

a) $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}$

$\Rightarrow \quad B \, 2\pi x = \mu_0 I \quad \Rightarrow B = \dfrac{\mu_0 I}{2\pi x}$

$$\vec{B} = \frac{\mu_0 I}{2\pi x} (-\hat{k})$$

b) $\phi_B = \int \vec{B} \cdot d\vec{A} = \displaystyle\int_r^{r+w} \frac{\mu_0 I}{2\pi x} \ell \, dx$

$$\Rightarrow \phi_B = \frac{\mu_0 I \ell}{2\pi} \ln\left(\frac{r+w}{r}\right)$$

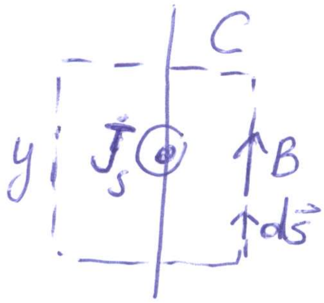
$|\varepsilon| = \dfrac{d\phi_B}{dt}$ and $\dfrac{dr}{dt} = v$

$$\frac{d\phi_B}{dt} = \frac{\mu_0 I \ell}{2\pi} \left[\frac{\overbrace{dr/dt}^{v}}{r+w} - \frac{\overbrace{dr/dt}^{v}}{r} \right] = \frac{\mu_0 I \ell}{2\pi} \left(\frac{-vw}{(r+w)r} \right)$$

$$\Rightarrow \varepsilon = -\frac{d\phi_B}{dt} = \frac{\mu_0 I \ell}{2\pi} \frac{vw}{r(r+w)} \quad \Rightarrow I = \frac{\varepsilon}{R} \text{ clockwise direction.}$$

Problem #3

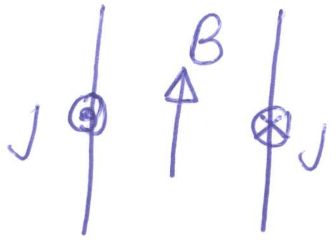
For a single sheet, choose a rectangular Amperian loop.



$$\oint_C \vec{B}\cdot d\vec{s} = \mu_0 J_s \cdot y$$

$$B\cdot 2y = \mu_0 J_s y \Rightarrow B = \frac{1}{2}\mu_0 J_s$$

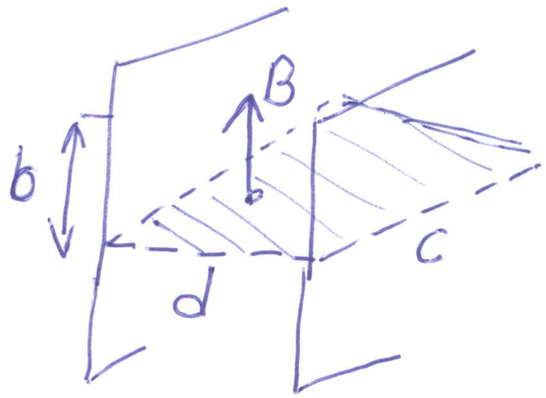
For two sheets we use SUPERPOSITION



$B=0$ outside the sheet

$\boxed{B=\mu_0 J_s}$ between the sheets

(b) Consider a surface betwen the sheets shown in the figure



magnetic flux

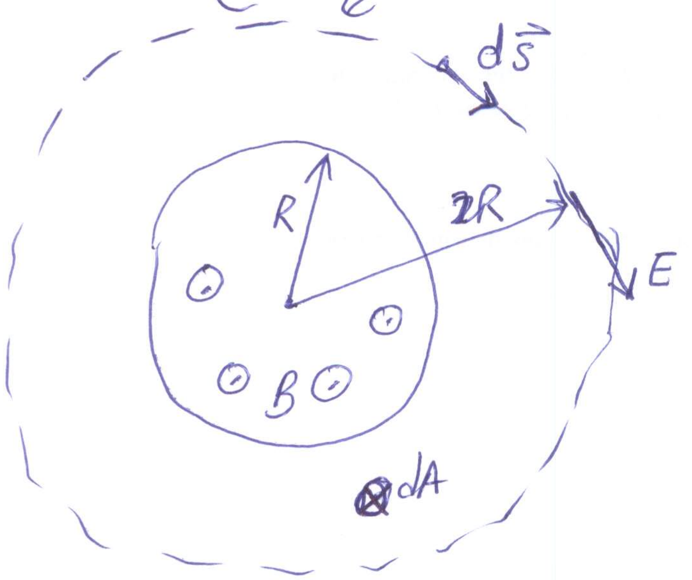
$$\Phi = \int \vec{B}\cdot d\vec{A} = Bcd$$

$$\phi = \mu_0 J_s c d$$

$$L = \frac{\Phi}{i} = \frac{\mu_0 J_s cd}{J_s \cdot b} = \mu_0 \frac{c\cdot d}{b}$$

Problem #4

Draw a circular loop of radius $r=2R$ centered at the origin and apply Faraday's law



$$\oint_C \vec{E}\cdot d\vec{s} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A}$$

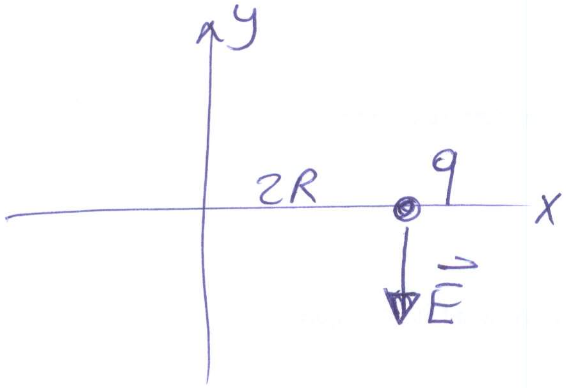
$$E\cdot 2\pi r = -\frac{d}{dt}\left[-B\pi (R)^2\right]$$

$r=2R$

$$\Rightarrow \quad E = \frac{R}{4}\frac{dB}{dt} = \frac{kR}{4}$$

(Electric field lines are circles. Electric field vectors are tangent to the circles in the clockwise direction)

The force on q is



$$\vec{F} = q\vec{E} = q\cdot\left(-\frac{kR}{4}\hat{j}\right)$$

$$\vec{F} = -\frac{kqR}{4}\hat{j}$$

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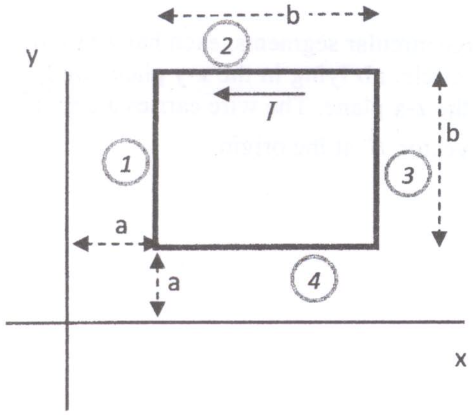

Q5 (5pt). A square loop of wire is placed in the **x-y** plane as shown in the figure. Side 1 and side 3 are parallel to y-axis, side 2 and side 4 are parallel to x-axis. The loop carries a current I. A magnetic field $\vec{B} = Cx\hat{i} - Cz\hat{k}$ is applied. Find the force acting on each side of the loop (side 1, side 2, side 3, and side 4).

$$① \quad x = a, \quad z = 0 \Rightarrow \vec{B}_1 = Ca\hat{i} \Rightarrow \vec{F}_1 = I \cdot \overbrace{b}^{\text{length}} \cdot \overbrace{Ca}^{\text{Magnetic Field}} [-\hat{j} \times \hat{i}]$$

$$\boxed{\vec{F}_1 = IabC\hat{k}}$$

$$② \quad y = a+b, \quad z = 0 \Rightarrow \vec{B}_2 = Cx\hat{i} \Rightarrow d\vec{F}_2 = I \cdot dx \cdot Cx\,[-\hat{i} \times \hat{i}] = 0$$

$$\Rightarrow \boxed{\vec{F}_2 = 0}$$

$$③ \quad x = a+b, \quad z = 0 \Rightarrow \vec{B}_3 = C(a+b)\hat{i} \Rightarrow \vec{F}_3 = I \cdot b \cdot C(a+b) \cdot [\hat{j} \times \hat{i}]$$

$$\boxed{\vec{F}_3 = -I(a+b)bC\hat{k}}$$

$$④ \quad y = a, \quad z = 0 \Rightarrow \vec{B}_4 = Cx\hat{i} \quad d\vec{F}_4 = I\,dx\,Cx\,[\hat{i} \times \hat{i}] = 0$$

$$\boxed{\vec{F}_4 = 0}$$

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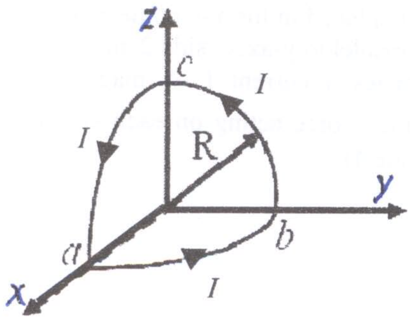

Q6 (5pt). A wire is bent into three circular segments, each has a radius R. Each segment is a quadrant of a circle, **ab** lying in the **x-y** plane, **bc** lying in the **y-z** plane, and **ca** lying in the **z-x** plane. The wire carries a current I. Calculate the net magnetic field vector $\vec{B}$ at the origin.

Biot-Savart for a full circular loop

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2} \Rightarrow dB = \frac{\mu_0 I}{4\pi R^2} dl$$

$$B = \int_{loop} dB = \frac{\mu_0 I}{4\pi R^2} \underbrace{\int dl}_{2\pi R}$$

$$B = \frac{\mu_0 I}{2R} \quad \perp \text{ to loop}$$

quarter loop on x-y plane:

$$\vec{B}_{ab} = \frac{1}{4} \cdot \frac{\mu_0 I}{2R} \hat{k}$$

quarter loop on y-z plane:

$$\vec{B}_{bc} = \frac{1}{4} \cdot \frac{\mu_0 I}{2R} \hat{\imath}$$

quarter loop on z-x plane:

$$\vec{B}_{ca} = \frac{1}{4} \cdot \frac{\mu_0 I}{2R} \hat{\jmath}$$

$$\Rightarrow \vec{B} = \vec{B}_{ab} + \vec{B}_{bc} + \vec{B}_{ca} = \boxed{\frac{\mu_0 I}{8R} (\hat{\imath} + \hat{\jmath} + \hat{k})}$$

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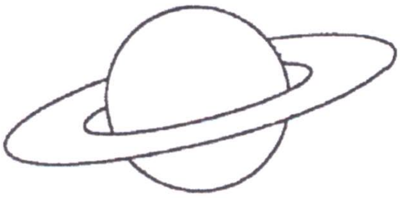
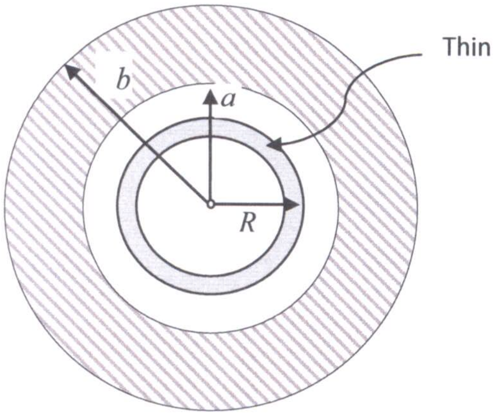


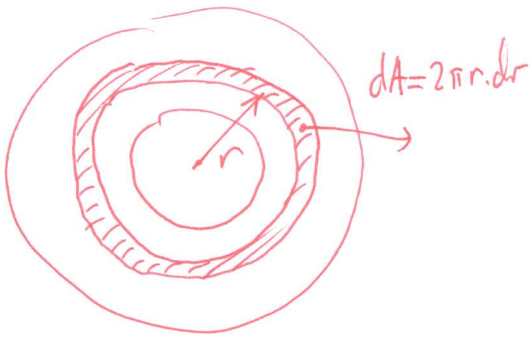
Q7 (5pt). A uniformly charged (Q) thin spherical shell of radius R is placed concentrically inside a uniformly charged (–Q) annulus of inner radius a and outer radius b. Find the electric potential at the center of the spherical shell. Take $V_\infty = 0$ if needed.

(Note : An annulus is a disk with a hole at its center).



Potential is a scalar quantity.
The contributions from the spherical shell and annulus will be calculated separately, then they will be added.
We take $V_\infty = 0 \Rightarrow dV = k \cdot dq / r$

Spherical Shell: $$V_{shell} = k \int \frac{dq}{R} = \frac{k}{R} \underbrace{\int_{shell} dq}_{+Q} = + \frac{kQ}{R}$$



Annulus: $$\sigma = \frac{Q_{annulus}}{\text{Area Annulus}} = \frac{-Q}{\pi(b^2-a^2)}$$

$$dV_{ring} = k \frac{dq_{ring}}{r} = k \frac{\sigma . dA}{r}$$

$$= - \frac{kQ \cdot 2\cancel{\pi}\cancel{r} \cdot dr}{\cancel{\pi}(b^2-a^2)\cancel{r}}$$

$$= - \frac{2kQ}{(b+a)(b-a)} \cdot dr$$

$$V_{annulus} = \int dV_{ring} = - \frac{2kQ}{(b+a)(b-a)} \int_a^b dr = - \frac{2kQ}{(b+a)\cancel{(b-a)}} \cancel{(b-a)}$$

$$V_{annulus} = - \frac{2kQ}{b+a}$$

$$\Rightarrow V = V_{shell} + V_{annulus} = \boxed{kQ\left[\frac{1}{R} - \frac{2}{a+b}\right]}$$

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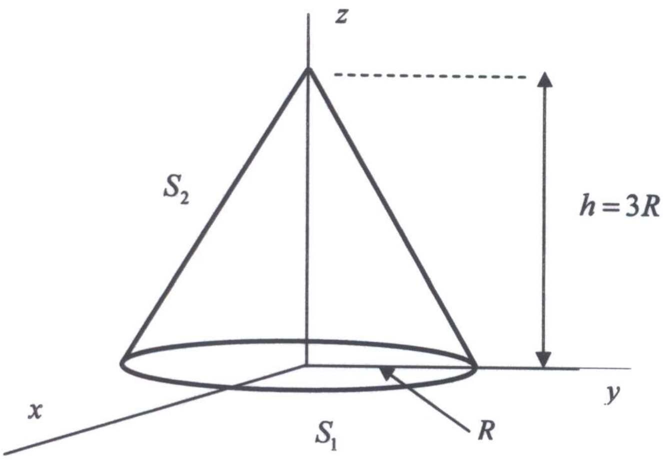


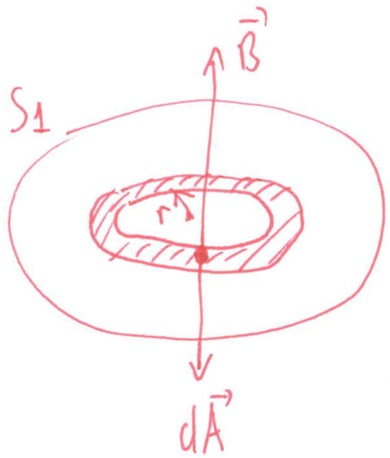
Q8 (5pt). A cone has a base radius R and height $3R$. The center of the base of the cone is located at the origin and the axis of the cone lies along the z-axis as shown in the figure. A non-uniform magnetic field $\vec{B} = \frac{B_0}{R} r\hat{k}$ present everywhere in space. r is the axial distance from the z axis ($r = \sqrt{x^2 + y^2}$).

(a) Calculate the magnetic flux through the base of the cone (surface S_1).

(b) Calculate the magnetic flux through the side surface of the cone (surface S_2).

Note: Area vector must be pointing outward from the cone.

a)



$$d\vec{A} = 2\pi r\,dr\,(-\hat{k})$$

$$\vec{B} = \frac{B_0}{R} r\,\hat{k}$$

$$d\Phi_1 = \vec{B}\cdot d\vec{A} = -\frac{B_0}{R} r \cdot 2\pi r \cdot dr$$

$$\Phi_1 = \int_{S_1} d\Phi_1 = -\frac{B_0}{R} 2\pi \int_0^R r^2\,dr = -\frac{2\pi B_0}{R}\cdot\frac{R^3}{3}$$

$$\boxed{\Phi_1 = -\frac{2\pi B_0 R^2}{3}}$$

b) Gauss Law for $\vec{B}$ field:

$$0 = \oint \vec{B}\cdot d\vec{A} = \underbrace{\int_{S_1} \vec{B}\cdot d\vec{A}}_{} + \underbrace{\int_{S_2} \vec{B}\cdot d\vec{A}}_{} \Rightarrow \boxed{\Phi_2 = +\frac{2\pi B_0 R^2}{3}}$$

$$0 = \Phi_1 + \Phi_2$$