Midterm – July 20^{th}, 2011 – 90 Minutes

NO QUESTIONS – NO CALCULATORS – SHOW YOUR WORK CLEARLY

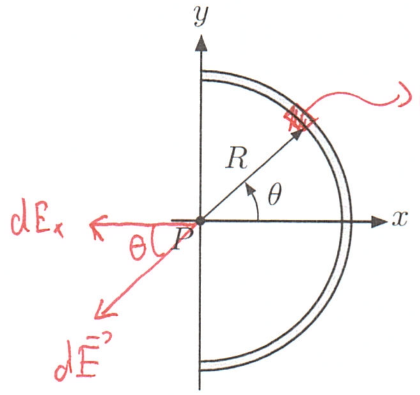
Phys No:	Name:

Question 1: *(Electric Fields) – 5 pts*

A line of charge is formed into a semicircle of radius R as shown in the figure. The charge per unit length along the semicircle is described by the expression $\lambda = \lambda_0 \cos\theta$. Calculate the total force vector on a point charge q placed at the center of curvature P.

[$\int \sin^2 x\, dx = \frac{x}{2} - \frac{\sin 2x}{4}$ $\int \cos^2 x\, dx = \frac{x}{2} + \frac{\sin 2x}{4}$ $\int \sin x \cos x dx = \frac{\sin^2 x}{2}$]

$\vec{E}$ is in the $-x$ direction alone due to symmetry.



$dq = \lambda \cdot R d\theta = \lambda_0 R \cos\theta . d\theta$

$$dE = k \frac{dq}{R^2}$$

$$dE_x = -k \frac{dq}{R^2} . \cos\theta = -k \frac{\lambda_0 R \cos\theta\, d\theta}{R^2} \cos\theta = -\frac{k\lambda_0}{R} \cos^2\theta\, d\theta$$

$$E_x = \int_{object} dE_x = -\frac{k\lambda_0}{R} \int_{-\pi/2}^{\pi/2} \cos^2\theta\, d\theta = -\frac{k\lambda_0}{R}\left[\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right]_{-\pi/2}^{\pi/2} = -\frac{k\lambda_0 \pi}{2R}$$

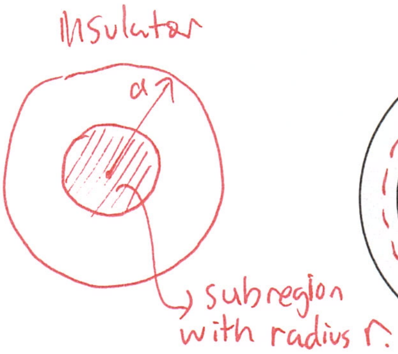
$$\Rightarrow \vec{E} = -\frac{k\lambda_0\pi}{2R}\,\hat{i} \quad \text{thus} \quad \vec{F} = q\vec{E} = -\frac{k\lambda_0 \pi q}{2R}\,\hat{i}$$

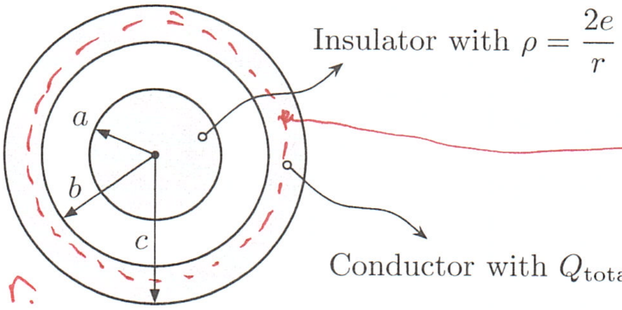
Question 2: *(Gauss's Law) – 5 pts*

Consider an insulating sphere with radius a and volumetric charge density $\rho = \frac{2e}{r}$ which is concentric with a conducting spherical shell with inner radius b, outer radius c and total charge $Q_{\text{total}} = 0$ as shown in the figure. (e is a constant.)

a) Find the surface charge densities σ_{in} and σ_{out} on the inner/outer surfaces of the conductor.

b) Find the electric field vector $\vec{E}$ for: $r<a$, $a<r<b$, $b<r<c$ and $r>c$.





$Q_{\text{insulator}}(r) = \int_{\text{shaded region}} dq = \int_{\text{s.r.}} \rho \cdot dV = \int_0^r \frac{2e}{r'} 4\pi r'^2 dr' = 4\pi r^2 \cdot e$

If we apply gauss law for a spherical surface inside conductor (where E=0)

$\Phi = \frac{Q_{in}}{\epsilon_0}$ ($\Phi = 0$) $\Rightarrow Q_{in} = 0 = Q_{\text{insulator}}(r=a) + \sigma_{in} 4\pi b^2$

$0 = 4\pi a^2 \cdot e + 4\pi b^2 \sigma_{in}$

$\boxed{\sigma_{in} = -e(a/b)^2}$

$Q_{\text{total,conductor}} = 0 = 4\pi b^2 \sigma_{in} + 4\pi c^2 \sigma_{out}$, with $\sigma_{in} = -e\cdot(a/b)^2$

$\Rightarrow \boxed{\sigma_{out} = (a/c)^2 \cdot e}$

$r<a$: $\Phi = \frac{Q_{in}}{\epsilon_0}$ $\quad 4\pi r^2 E = \frac{Q_{\text{insulator}}(r)}{\epsilon_0}$

$4\pi r^2 E = \frac{4\pi r^2 e}{\epsilon_0}$

$\vec{E} = \frac{e}{\epsilon_0}\hat{r}$

$r>c$: same as case $a<r<b$ because total-charge-in is the same ($Q_{\text{total\_conductor}} = 0$)

$\Rightarrow \vec{E} = \frac{a^2 e}{\epsilon_0 r^2}\hat{r}$

$a<r<b$: $\Phi = \frac{Q_{in}}{\epsilon_0} \Rightarrow 4\pi r^2 E = \frac{Q_{\text{insulator}}(r=a)}{\epsilon_0}$

$4\pi r^2 E = 4\pi a^2 e/\epsilon_0$

$\vec{E} = \frac{a^2 e}{\epsilon_0 r^2}\hat{r}$

$b<r<c$

$E=0$ inside conductor.

Phys No:	Name:

Question 3: *(Electric Potential) – 5 pts*

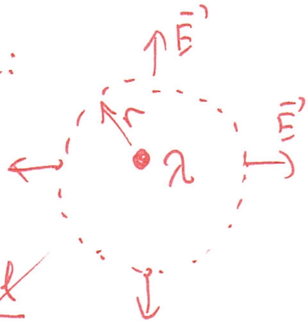
An infinitely long line charge having a uniform charge per length λ is located along the z axis as shown in the figure.

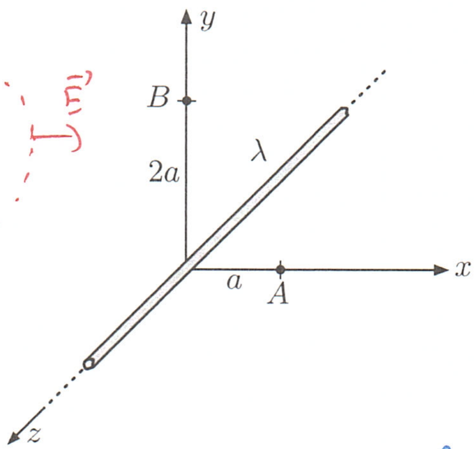
a) Find the potential difference $\Delta V = V_B - V_A$ between the points $A = (a, 0, 0)$ and $B = (0, 2a, 0)$.

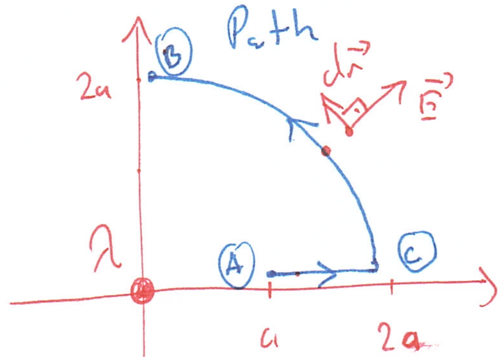
b) Find the necessary work to move a point charge q from point A to point B.

Electric Field:

$$\Phi = \frac{Q_{in}}{\epsilon_0}$$







$$E \cdot 2\pi r \cdot \ell = \frac{\lambda \cdot \ell}{\epsilon_0}$$

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0} \cdot \frac{1}{r}\hat{r}$$

Because $\vec{E}$ is perpend. to $d\vec{r}$ for the arc.

$$\Delta V = V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{r} = -\int_A^C \vec{E}\cdot d\vec{r} + \int_C^B \vec{E}\cdot d\vec{r} \quad (= 0)$$

$$= -\int_a^{2a} \left(\frac{\lambda}{2\pi\epsilon_0}\cdot\frac{1}{r}\hat{r}\right)\cdot(dr\,\hat{r}) = -\frac{\lambda}{2\pi\epsilon_0}\int_a^{2a}\frac{dr}{r} = -\frac{\lambda}{2\pi\epsilon_0}\ln\frac{2a}{a} = -\frac{\lambda\ln 2}{2\pi\epsilon_0}$$

$$\Rightarrow \boxed{\Delta V = -\frac{\lambda \ln 2}{2\pi\epsilon_0} = -2\cdot k\cdot\lambda\cdot\ln 2}$$

$$\Rightarrow \boxed{W = q\cdot\Delta V = -2k\lambda q\ln 2}$$ Work done by external agent.

or $$\boxed{W = -q\cdot\Delta V = 2k\lambda q\ln 2}$$ Work done by the field

Phys No:	Name:

Question 4: *(Capacitance and Dielectric) – 5 pts*

Consider a two-capacitor system which includes two identical parallel plate capacitors each charged with Q and has a capacitance of C as shown in Figure 1. Then, we insert a material with dielectric coefficient κ tightly into one of the capacitor as shown in Figure 2. Find Q_1 and Q_2, the charges in each capacitors, in terms of Q and κ after we insert the dielectric material.

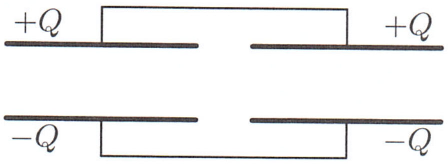


Figure 1

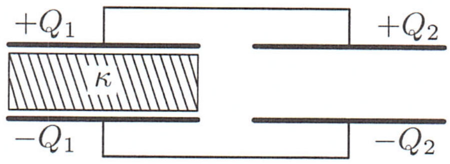


Figure 2

(1) Charge is a conserved quantity. Total charges on upper plates:

$$\Rightarrow \underbrace{Q + Q}_{\text{Initial}} = \underbrace{Q_1 + Q_2}_{\text{Final}}$$

(2) Potential difference of the capacitors must be the same for both capacitors in the final state.

$$\Delta V^* = \frac{Q_1}{C_{left}} = \frac{Q_2}{C_{right}} \Rightarrow \frac{Q_1}{\kappa \cdot C} = \frac{Q_2}{C}$$

$$Q_1 = \kappa Q_2$$

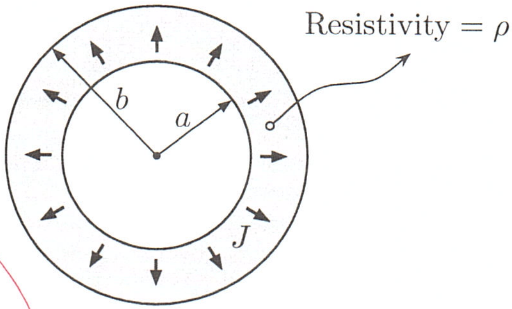
Combine these two equations:

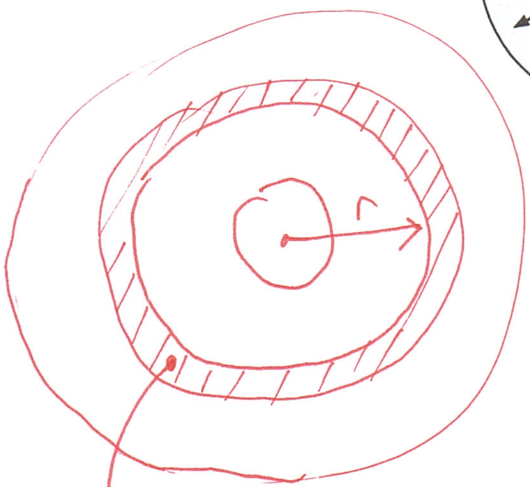
$$\left.\begin{array}{l} Q_1 + Q_2 = 2Q \\ -Q_1 + \kappa Q_2 = 0 \end{array}\right\} \Rightarrow \boxed{Q_1 = \frac{\kappa}{\kappa+1} 2Q \qquad Q_2 = \frac{1}{\kappa+1} 2Q}$$

Phys No:	Name:

Question 5: *(Current and Resistance) – 5 pts*

A spherical shell with inner radius a and outer radius b is formed from a material of resistivity ρ. It carries current radially, with uniform density in all directions. Find the resistance of the system.





$$dR = \rho \frac{dr}{4\pi r^2}$$

dr → penetration distance

$4\pi r^2$ → total area of the dR through which J passes

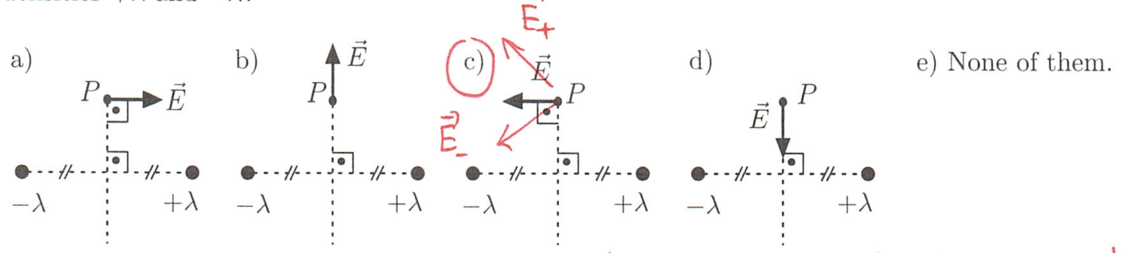
$$R = \int_{shell} dR = \frac{\rho}{4\pi}\int_a^b \frac{dr}{r^2} = \boxed{\frac{\rho}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right)}$$

Because, can be considered to be collection of series resistances.

Question 6: *(Conceptual Short Questions) – 5 pts*

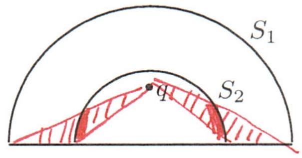
You will answer 5 conceptual questions each worth 1 point. Mark the correct choice and explain the reason very briefly in a single line by words alone. You get no credit if the explanation is missing or wrong.

1) Which one of the following is correct for the $\vec{E}$ at P due to two infinitely long parallel wires with uniform linear charge densities $+\lambda$ and $-\lambda$?



Explain: y-components for each contribution cancels each other: purely in $-\hat{x}$

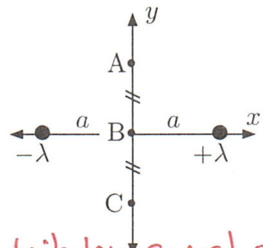
2) Consider two concentric hemispherical open surfaces S_1 and S_2 (bottoms are open). There is a point charge q located just below the smaller surface as shown in the figure. Φ_1 and Φ_2 are the electric flux due to charge q through the surfaces S_1 and S_2 respectively. Which one of the following is true?



a) $|\Phi_1| > |\Phi_2|$ b) $|\Phi_1| < |\Phi_2|$ c) $|\Phi_1| = |\Phi_2|$ d) None of them.

Explain: Field lines in the shaded region passes only through S_2. More lines; more Φ.

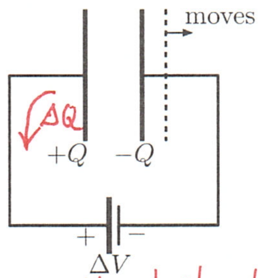
3) Consider two infinite wires parallel to the z-axis with uniform linear charge densities $+\lambda$ and $-\lambda$ as shown in the figure. The potentials at points A, B and C are V_A, V_B and V_C respectively. Which one of the following is true?



a) $V_A = V_C < V_B$ b) $V_A = V_C > V_B$ c) $V_A = V_B = V_C$ d) None of them.

Explain: Potential is a scalar quantity. For all three points, contributions cancel each other $\Rightarrow$ SAME.

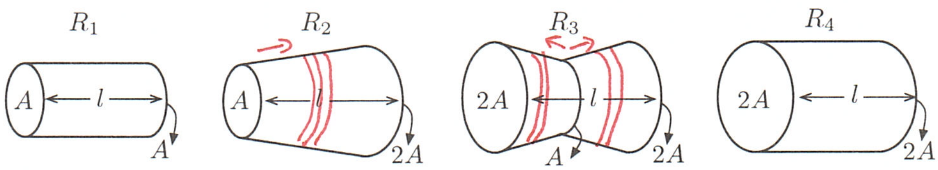
4) Consider a parallel plate capacitor as shown in the figure. One of the plate is free to move along the direction shown in the figure. If we pull that plate to the right as shown in the figure, determine the direction of current.



a) Clockwise b) Counter-clockwise c) $I = 0$.

Explain: As it moves, C decreases $\Rightarrow Q = \Delta V \cdot C$ decreases $\therefore$ charges on the left plane should decrease $\Rightarrow$ C.C.W.

5) The following resistances are made of identical materials with resistivity of ρ, and their total length are the same. Surface areas are indicated as seen in the figure. The current flows from left to right for each resistor with a uniform distribution over any circular cross section. Which one of the following is true?



a) $R_1 < R_2 < R_3 < R_4$ b) $R_4 < R_3 < R_2 < R_1$ c) $R_1 < R_2 = R_3 < R_4$ d) $R_4 < R_3 = R_2 < R_1$

Explain: $R \sim 1/A \Rightarrow (R_4 < R_1)$ and $(R_2 < R_1)$ and $(R_2 = R_3)$ same object; alternative order of disks.