Boğaziçi University
Department of Physics

Phys 311/407 Summer 2014

Problem Set #7

Reading: Rohlf Chapter 6

Useful integrals:

$$\int_{-\infty}^{\infty} e^{-(ax^2+bx)} = \sqrt{\frac{\pi}{a}} e^{b^2/4a} \qquad a > 0$$

$$\int_{-\infty}^{\infty} x^2 \cdot e^{-(x/a)^2} = \frac{a^3}{2}\sqrt{\pi} \qquad a > 0$$

Problem 1:
In class, we stated that $\Delta x \cdot \Delta p_x = \hbar/2$ for a Gaussian wave function by hand-waving. In this question you will show this quantitatively. Consider the wave function $\psi(x) = Ce^{-\frac{1}{2}(x/\sigma)^2}$ for a particle:

a) Find C such that the probability distribution is properly normalized,

b) Find $g(k)$,

c) Find the average position of the particle, $\langle x \rangle = \int \mathrm{d}P \cdot x$, where P is the probability function obeying $\mathrm{d}P/\mathrm{d}x = |\psi(x)|^2$

d) Find the average x^2; $\langle x^2 \rangle = \int \mathrm{d}P \cdot x^2$,

e) Find Δx; $(\Delta x)^2 = \int P \cdot (x - \langle x \rangle)^2$, (Hint: you do not to evaluate this integral; you can use the results found in previous parts.)

f) Using the symmetry between $\psi(x)$ and $g(k)$ functions, write down Δp_x without evaluating any integral,

g) Show that $\Delta x \cdot \Delta p_x = \hbar/2$.

Problem 2: – Rohlf 5.22 – Modified
Even a professional musician with "perfect pitch" will have trouble identifying the pitch of a note if the duration is too short. Why? State your reasoning clearly, draw sketch of the waves if needed.

Problem 3:
If a particle can exist either at $x = -L$ or at $x = L$ with equal probability, what is Δx?

Problem 4: – Rohlf 5.20 – Modified
Consider an electron and a proton each confined to a volume of 10^{-30} m^3. Which particle has a larger minimum kinetic energy? Why? State your reasoning clearly.