Boğaziçi University
Department of Physics

Phys 311/407 | Summer 2014

Problem Set #8

Reading: Rohlf 8.1, 8.2, 8.3

Problem 1:
Consider the wave packet $\psi_p(p) = C\exp(-a|p|/\hbar)$ where a is a positive real number. Find the normalization constant C such that:

$$\int |\psi_p(p)|^2 \, \mathrm{d}p = 1$$

Problem 2:
Find $\psi(x)$ for the momentum wave function $\psi_p(p)$ given in question 1. Show that it is normalized.

Problem 3:
Show that

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

for the wave functions given above. [Hint: $\int u^2 e^{-u} \, \mathrm{d}u = -(u^2 + 2u + 2)e^{-u}$]

Problem 4: – Rohlf 7.31 – Modified
Consider a non-relativistic particle of mass m that is *confined* by a potential: $V(x) = +\infty$ for $|x| > L/2$, and $V(x) = -V_0 < 0$ for $|x| \leq L/2$. (a) Determine the allowed energies of the particle. (b) If the particle is an electron, $L = 0.2$ nm and $V_0 = 20$ eV, how many states have a negative energy?

Problem 5: – Rohlf 7.23
Show by direct substitution into the Scrödinger equation that the wave function

$$\psi(x) = C\left(\alpha^{3/2}x^3 - \frac{3}{4}\sqrt{\alpha}x\right)e^{-\alpha x^2}$$

where $\alpha = m\omega/2\hbar$ and C is a normalization constant, is a solution of the harmonic oscillator, $V(x) = \frac{1}{2}m\omega^2 x^2$. Calculate the corresponding energy.