Boğaziçi University
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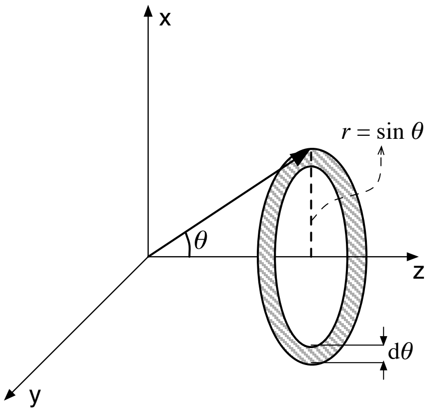
Phys 311/407 Summer 2014

Problem Set #5 Solutions

Answer 1:

a) The directions of the cosmic background radiation are uniformly distributed over the full solid angle. The rate of particles that pass through a unit area depends on the azimuthal angle θ: The maximum flux is obtained when $\theta = 0, \pi$, and the minimum flux (=0) is obtained when $\theta = \pi/2$. Thus, we, first, need to find the number density as a function of azimuthal angle θ. This can be found by using the solid angle for photon directions between θ and $\theta + \mathrm{d}\theta$:



The number density as a function of azimuthal angle must be proportional to the ratio of solid angle defined by the shaded area above to the total solid angle:

$$\bar{n}(\theta)\mathrm{d}\theta = k' \frac{2\pi r \, \mathrm{d}\theta}{4\pi} = k' \frac{2\pi \sin(\theta)\mathrm{d}\theta}{4\pi}$$

where k' is the normalization constant that yields the total number density of n over the full 4π solid angle coverage. Since, k' is unknown, let us define another constant k such that it includes all the other constants as follows:

$$\bar{n}(\theta)\mathrm{d}\theta = k \sin(\theta)\mathrm{d}\theta$$

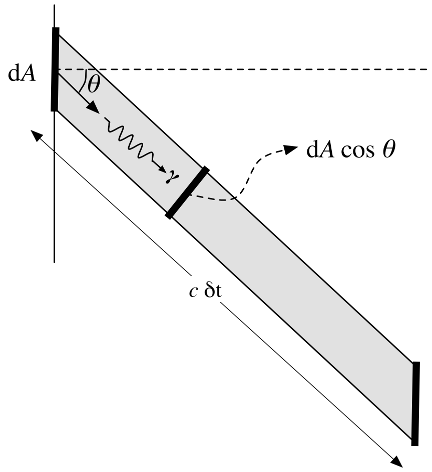
Then, we write the total number density in order to find k:

$$n = \int_0^\pi \bar{n}(\theta)\mathrm{d}\theta = k \int_0^\pi \sin(\theta)\mathrm{d}\theta = 2k$$

which gives the normalization: $k = n/2$. Thus, we get the number density for the cosmic background radiation in the direction between $[\theta, \theta + \mathrm{d}\theta]$ as:

$$\bar{n}(\theta)\mathrm{d}\theta = \frac{n}{2} \sin(\theta)\mathrm{d}\theta$$

This was the first step in solving this problem. Now, we can find the particles that pass through an area of $\mathrm{d}A$ with an azimuthal angle between $[\theta, \theta + \mathrm{d}\theta]$, using the number density for given direction, and integrate it over the half range for which the number of particles that pass through the area from one side to the other is found. Then, we multiply it by two –using the symmetry– to find the total rate for both sides.



The photons that pass through the area dA, with the azimuthal angle $[\theta, \theta + \mathrm{d}\theta]$ in time δt occupy the volume:

$$\mathrm{d}V = \mathrm{d}A\,|\cos\theta|\,c\,\delta t$$

Then, the number of particles that pass through dA in $\delta t = 1$ second with the azimuthal angle $[\theta, \theta + \mathrm{d}\theta]$ can be calculated using the angular number density we found:

$$\begin{aligned} \mathrm{d}(\mathrm{d}N) &= \mathrm{d}V \cdot \bar{n}(\theta)\mathrm{d}\theta \\ &= \mathrm{d}V \cdot \frac{n}{2}\sin\theta\,\mathrm{d}\theta \\ &= \mathrm{d}A\,\frac{n}{2}\,c\,|\cos\theta|\sin\theta\;\mathrm{d}\theta \end{aligned}$$

The integration over the total area A yields:

$$\mathrm{d}N = \frac{Acn}{2}|\cos\theta|\;\sin\theta\;\mathrm{d}\theta$$

The number of particles that pass through area A from left to right with respect to the above sketch is calculated by the following integral:

$$N_{\text{left}\to\text{right}} = \int_0^{\pi/2} \mathrm{d}N = \frac{Acn}{2}\int_0^{\pi/2}\cos\theta\sin\theta\;\mathrm{d}\theta = \frac{Acn}{4}$$

By symmetry, $N_{\text{left}\to\text{right}} = N_{\text{right}\to\text{left}}$, then the total number of particles that pass through a total area of A per second can be found by:

$$\begin{aligned} N &= \frac{Acn}{2} \\ &= \frac{(1.8 \times 0.4\ \mathrm{m}^2)\cdot(3\times 10^8\ \mathrm{m}/sec)\cdot(4\times 10^8\ 1/\mathrm{m}^3)}{2} \\ N &= 4.3\times 10^{16}\ \mathrm{sec}^{-1} \end{aligned}$$

This is about 2–3 orders of magnitude smaller than the number of photons emitted by the laser in question 2.

b)

$$E = u \cdot V = (2.7\times 10^5\ \mathrm{eV}/m^3)\cdot(1.8\times 0.4\times 0.3\ \mathrm{m}^3) = 39\ \mathrm{ke}V$$

Answer 2:

Energy of a single photon for this green laser is:

$$\begin{aligned} E_\gamma &= \frac{hc}{\lambda} = \frac{6.6\times 10^{-34}\ \mathrm{J}\cdot s \times 3\times 10^8\ \mathrm{m}/s}{532\times 10^{-9}\ \mathrm{m}} \\ &= 3.7\times 10^{-19}\ \mathrm{J} \end{aligned}$$

Power is:

$$P = \frac{N \cdot E_\gamma}{\Delta T}$$

Thus N is:

$$\begin{aligned} N &= P \cdot \Delta T / E_\gamma \\ &= (5 \text{ J}/s)(1 \text{ s})/(3.7 \times 10^{-19} \text{ J}) \\ &= 1.4 \times 10^{19} \text{ photons per second.} \end{aligned}$$

Answer 3:
The energy of directed photon is:

$$E_\gamma = \frac{hc}{\lambda} = \frac{1240 \text{ eV} \cdot nm}{200 \text{ nm}} = 6.2 \text{ eV}$$

The work function for the silver metal is $\phi = 4.3$ eV. Then, the maximum kinetic energy of the emitted electron from the silver surface is, simply:

$$E_k^{max} = E_\gamma - \phi = 6.2 - 4.3 = 1.9 \text{ eV}$$

Answer 4:
The orbital frequency is found by the speed divided by the orbital path length:

$$f_{orb} = \frac{v_n}{2\pi r_n} \qquad v_n = \frac{n\hbar}{mr_n} \qquad r_n = \frac{n^2\hbar^2}{mke^2}$$

and, combining these three equations and using the definition of fine structure constant $\alpha = ke^2/\hbar c$, we get:

$$f_{orb} = \frac{\alpha^2 mc^2}{h} \cdot \frac{1}{n^3}$$

The orbital frequency at $n = 3$:

$$f_{orb,3} = \frac{\alpha^2 mc^2}{h} \cdot \frac{1}{27}$$

The frequency of photon released by the transition from n_i to n_f is:

$$\begin{aligned} E_\gamma = h \cdot r_{rad} &= \frac{\alpha^2 mc^2}{2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \\ f_{rad} &= \frac{\alpha^2 mc^2}{2h}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \end{aligned}$$

The first allowed transition is $n = 3 \to 2$:

$$f_{rad} = \frac{\alpha^2 mc^2}{2h}\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{\alpha^2 mc^2}{h} \cdot \frac{5}{72}$$

The ratio of frequencies:

$$f_{orb}/f_{rad,3\to2} = \frac{1}{27} \cdot \frac{72}{5} \simeq 0.5$$

The second allowed transition is $n = 3 \to 1$:

$$f_{rad} = \frac{\alpha^2 mc^2}{2h}\left(\frac{1}{1} - \frac{1}{9}\right) = \frac{\alpha^2 mc^2}{h} \cdot \frac{4}{9}$$

The ratio of frequencies:

$$f_{orb}/f_{rad,3\to1} = \frac{1}{27} \cdot \frac{9}{4} \simeq 0.1$$

Answer 5:
We can use $m_p \gg m_\mu$ (ratio is ~1:10),

$$E_n = -\frac{\alpha^2 m_\mu c^2}{2} \cdot \frac{1}{n^2}$$

In order to have the minimum wavelength, the energy must be the maximum; the transition is for $n = \infty \to 1$.

$$\begin{aligned} E_\gamma = E_\infty - E_1 &= 0 + \frac{\alpha^2 m_\mu c^2}{2} \\ &= (1/137)^2 \cdot (106 \text{ Me}V/c^2) \cdot (c^2)/2 \\ &= 2800 \text{ e}V \end{aligned}$$

Then, the wavelength is found as:

$$\lambda = \frac{1240 \text{ e}V \cdot nm}{E_\gamma} = \frac{1240 \text{ e}V \cdot nm}{2800 \text{ e}V} \simeq 0.5 \text{ n}m$$