Boğaziçi University
Department of Physics

Phys 311/407 Summer 2015

Problem Set #1 Solutions

Answer 1:

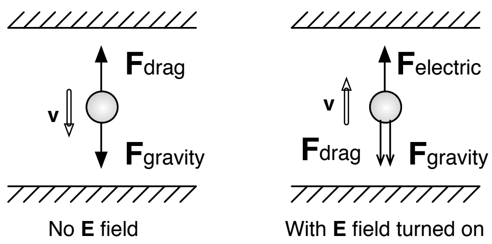
$$\begin{aligned} d_{\text{earth}} &\approx \frac{40.000\text{km}}{\pi} = \frac{4 \times 10^7\text{m}}{\pi} \approx 10^7\text{m} \\ d_{\text{apple}} &\approx 10\text{cm} = 0.1\text{m} \\ d_{\text{H}} &\approx 3 \times 10^{-10}\text{m} \end{aligned}$$

a) $\text{Ratio}_1 = \frac{d_{\text{earth}}}{d_{\text{apple}}} \approx \frac{10^7}{10^{-1}} = 10^8$

b) $\text{Ratio}_2 = \frac{d_{\text{apple}}}{d_{\text{H}}} \approx \frac{10^{-1}}{3\times 10^{-10}} = 3 \times 10^8$

$\Longrightarrow$ These ratios are comparable; they have the *same order of magnitudes.*

Answer 2:



By definition, the speed of particle where E-field is turned on is the same as the one without E-field. So, $\left|\vec{F}_{\text{drag}}\right|$'s seen above figures are the same, which only depends on the speed of the particle in a media, and its direction is opposite to the direction of the velocity.

No-field case: ($+y$ direction is upward)

$$\begin{aligned} \vec{F}_{\text{total}} &= \vec{F}_{\text{drag}} + \vec{F}_{\text{gravity}} \\ F_y &= F_{\text{drag}} - mg \quad F_y = 0 \text{ because it travels with constant speed} \\ 0 &= F_{\text{drag}} - mg \;\Rightarrow\; F_{\text{drag}} = mg \end{aligned}$$

Electric field is turned on:

$$\begin{aligned} \vec{F}_{\text{total}} &= \vec{F}_{\text{drag}} + \vec{F}_{\text{gravity}} + \vec{F}_{\text{electric}} \\ F_y &= F_{\text{electric}} - mg - F_{\text{drag}} \\ 0 &= F_{\text{electric}} - mg - F_{\text{drag}} \end{aligned}$$

$$
\begin{aligned}
0 &= F_{\text{electric}} - mg - mg \\
F_{\text{electric}} &= 2mg \\
q \cdot E_y &= 2mg \\
-e \cdot E_y &= 2mg \\
E_y &= -\frac{2mg}{e} = -\frac{2 \cdot 10^{-14}\text{kg}\, 9.8\,\text{m/sec}^2}{1.6 \cdot 10^{-19}\,\text{C}} \\
E_y &= -1.225 \cdot 10^6\ \text{V/m}
\end{aligned}
$$

Note that **a)** "–" sign indicates that the electric field must be downwards, **b)** this indicates that one needs about 100 kV of voltage for a reasonably separated ($\approx$ 10 cm) set of metal plates to create this much electric field.

Answer 3:
Conversion factors:

See the inner cover page of Rohlf (or any other modern physics books)

$$1\,\text{u} = 1.66 \times 10^{-27}\,\text{kg} = 931.5\,\text{MeV/c}^2$$

$$\Rightarrow \quad 1\,\text{kg} = 5.6 \times 10^{29}\,\text{MeV/c}^2$$

Then:

$$m_e = 9.11 \times 10^{-31}\,\text{kg} = 9.11 \times 10^{-31}\,\text{kg} \ \times 5.6 \times 10^{29}\,\frac{\text{MeV}}{\text{c}^2}\,\frac{1}{\text{kg}}$$

$$\boxed{m_e = 0.511\,\text{MeV/c}^2}$$

$E = m \cdot c^2 = 0.511\,\frac{\text{MeV}}{c^2} \cdot c^2$

$$\boxed{E = 0.511\ \text{MeV}}$$

Answer 4:
Well, the intuition tells the mass energy of any macroscoping object must be greater than the kinetic energy of another macroscoping object at non-relativistic speeds due to the fact that $c^2 \sim 10^{17}$ (or $\frac{v^2}{c^2} \ll 1$) provided that the ratio of the masses of these two objects is smaller than this huge number. Let's work out the estimations to see if this holds for the given example.

Estimation of the mass of a 747 jumbo jet (the giant airplane made by Boeing):

The mass of a trailer-track is $\sim$ 10 tons (remember the traffic signs before small bridges). The mass of 747 jumbo jet must be an order of magnitude (≥ 10) greater than the mass of a regular trailer-track.

$$\Rightarrow \quad m_{747} \sim 100\text{–}200\,\text{tons} \approx 1\text{–}2 \times 10^5\,\text{kg}$$

Estimation of the cruising speed:

It travels slower than speed of sound ($v_{\text{sound}} \approx 300$ m/s), but pretty close to that speed.

$$\frac{2}{3} v_{\text{sound}} < v_{747} < v_{\text{sound}} \quad \Rightarrow \quad v_{747} \sim 250\,\text{m/s}$$

$$E_{K,747} = \frac{1}{2}\, m_{747}\, v_{747}^2$$

$$\boxed{E_{747} \sim\ 10^{10}\,\mathrm{J}}$$

Estimation of the mass of a mosquito:

$$m_{\rm bug} \approx 1 \text{ gr} = 10^{-3} \text{ kg}$$

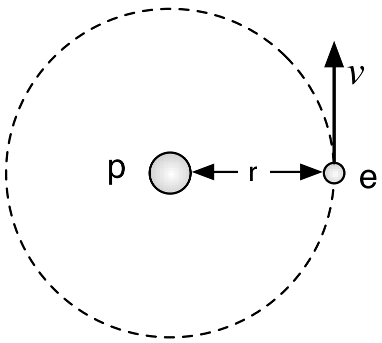
Then:

$$E_{\rm bug} = m\, c^2 \approx 10^{-3} \times (3 \cdot 10^8)^2 \text{ J}$$

$$\boxed{E_{bug} \sim 10^{14}\,\mathrm{J}}$$

$\Longrightarrow$ Although there may be up to an order of magnitude error in our estimations, the mass energy of a mosquito is about 4 orders of magnitudes greater than the kinetic energy of a jumbo jet at cruising speeds.

Answer 5:



a) Let us find the attractive force for the case of EM force:

$$d_H \sim 3 \times 10^{-10}\,\mathrm{m} \quad r_H \approx 1.5 \times 10^{-10}\,\mathrm{m}$$

$$\begin{aligned} F_{\mathrm{EM},\hat{r}} = \frac{k\, q_e\, q_p}{r^2} &= -\frac{9 \cdot 10^9 \cdot (1.6 \cdot 10^{-19})^2}{1.5 \cdot (10^{-10})^2}\,\mathrm{N} \\ &\sim -10^{-8}\,\mathrm{N} \end{aligned}$$

Then, this attractive force is equal to the gravitational force for a gravitational atom:

$$\begin{aligned} F_{\mathrm{gravity},\hat{r}} &= -\frac{G\, m_e\, m_p}{r^2} = -10^{-8}\,\mathrm{N} \\ r &= \left[\frac{G\, m_e\, m_p}{10^{-8}\,\mathrm{N}}\right]^{\frac{1}{2}} \\ &= \left[\frac{6.7 \times 10^{-11} \times 9.1\, 10^{-31} \times 1.7 \times 10^{-27}}{10^{-8}}\right]^{\frac{1}{2}} \,\mathrm{m} \end{aligned}$$

$$\boxed{r \approx 3 \times 10^{-30} \text{ m}}$$

b) We use the fact that $m_p \gg m_e$ such that the proton is stationary in such a system. Then, the kinetic energy can be found using the equation of centripetal force:

$$\begin{aligned} F_{\text{gravity},\hat{r}} &= F_{c,\hat{r}} = -\frac{m_e\, v^2}{r} \\ E_{\text{K}} &= \frac{1}{2}\, m_e\, v^2 = -\frac{1}{2}\, r\, F_{\text{gravity},\hat{r}} = \frac{3 \times 10^{-30} \times 10^{-8}}{2} \text{ J} \end{aligned}$$

$$\boxed{E_{\text{K}} \approx 1.5 \times 10^{-38} \text{ J}}$$

This is an incredibly small amount of energy; tens of orders of magnitudes smaller than eV which is *about* the order of magnitude of the binding energy of the electron of a hydrogen atom. Also, note that the radius we found for a gravitational atom is much smaller than the dimension of the structure of proton itself. Conclusion: gravitational force is so weak compared to EM force, we can not have such a system.