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Problem Set #4 Solutions

Answer 1:

First, we need to estimate the outside temperature on South Pole. The temperature in Siberia is around 40–20 degrees celsius below zero, take another 30°C for the poles, and say it is $T_{out} \approx -60$°C. Then,

$$\begin{aligned} R_{emit} &= \sigma T_{\text{body}}^4 \\ R_{absorb} &= \sigma T_{\text{T}_{out}}^4 \\ R &= R_{emit} - R_{absorb} \\ P/A &= (T_{\text{body}}^4 - T_{\text{out}}^4) \end{aligned}$$

where A is the total area of the body, which is roughly 2 m^2.

a)

$$\begin{aligned} P &\approx A\sigma(T_{\text{body}}^4 - T_{\text{out}}^4) \\ &\approx 2 \times 5.7 \times 10^{-8}(311^4 - 213^4) \text{ W} \\ &\approx 832 \text{ W} \end{aligned}$$

b)

$$\begin{aligned} P &\approx A\sigma(T_{\text{body}}^4 - T_{\text{room}}^4) \\ &\approx 2 \times 5.7 \times 10^{-8}(311^4 - 290^4) \text{ W} \\ &\approx 260 \text{ W} \end{aligned}$$

Thus, the body looses energy three times slower in the room compared to the loss happens outside.

c) Here, we need to estimate the total kinetic energy of the molecules inside the body. First, to the first order approximation, body is made of water; water molecules. Then, we need to approximate the average kinetic energy of each molecule. As you have seen in Phys 221, each degree of freedom introduces $\frac{1}{2}kT$ kinetic energy. Let us count the number of degrees of freedom:

3	:	for center of mass kinetic energy in 3 dimensions
2	:	for vibrational kinetic energy of two bounds of H_2O
3	:	for rotational kinetic energy around 3 axes

Thus the total average kinetic energy of a water molecule can be approximated by:

$$\langle E \rangle = (3 + 2 + 3)\frac{1}{2}kT = 4kT$$

The total numer of water molecules is $N = m/m_{H_2O}$. Take the weight of an adult body to be about 80 kg, thus:

$$N = \frac{80 \text{ kg}}{18u} = 2.6 \times 10^{27}$$

Then, the total kinetic energy can be approximated[1] as:

$$E = 4NkT = 4 \times 2.6 \times 10^{27} \times 1.38 \times 10^{-23} \times 311 = 45 \text{ MJ}$$

This amount of energy is about 4, 5 orders of magnitude greater than the energy loss per second in parts a) and b) respectively. Nevertheless, within hours ($\sim 10^4$ seconds) the body looses appreciable amount

[1] Note that this is a **roug**h approximation. The total energy is likely to be more than this because 1) water is not an ideal gas, 2) there is also potential energy of the bounds.

of energy such that the body temperature drops dangerously to fatal levels[2] assuming that there is no internal source of energy. Note that this person dies in a shorter amount of time ($\sim 3\times$, when there is no internal source of energy) when he stays outside.

Answer 2:

$$
\begin{aligned}
T_{\text{tea}} &= 373\ \text{K} \\
T_{\text{room}} &= 300\ \text{K} \\
A_{glass} &= 10\ \text{cm} \times 10\ \text{cm} = 10^{-2}\ \text{m}^2 \\
V_{glass} &= 10\ \text{cm} \times 3\ \text{cm} \times 3\text{cm} \approx 100\ \text{cm}^3 = 10^{-4}\ \text{m}^3 \\
m &= \rho_{water} \times V_{glass} = 0.1\ kg
\end{aligned}
$$

The assumption is that the change in temperature is small such that we assume a constant change in energy per unit area per unit time:

$$
\begin{aligned}
R &= \sigma\,(T_{\text{tea}}^4 - T_{\text{room}}^4) \\
P_{\text{emit}}/A &= \sigma\,(T_{\text{tea}}^4 - T_{\text{room}}^4) \\
P_{\text{emit}} &= A\,\sigma\,(T_{\text{tea}}^4 - T_{\text{room}}^4) \\
\Delta E_{\text{emit}}/\Delta t &= A\,\sigma\,(T_{\text{tea}}^4 - T_{\text{room}}^4) \\
\Delta E_{\text{emit}} &= A\,\sigma\,(T_{\text{tea}}^4 - T_{\text{room}}^4)\Delta t \\
&= 10^{-2} \times 5.7 \times 10^{-8} \times (373^2 - 300^2) \times 900\ \text{J} \\
&= 5700\ \text{J}
\end{aligned}
$$

Full credit will be granted to those who use $E \approx$ 4-5NkT. However, let us find a better approximation. The specific heat (capacity) of water is known; $c = 4180$ J$/kg \cdot K$, which is defined to be the change in energy per unit mass to increase (decrease) the temperature by 1 K, such that (Phys 102):

$$\Delta Q = cm\Delta T$$

For $\Delta E = \Delta Q + \overbrace{\Delta W}^{=0} \;\rightarrow\; \Delta Q = \Delta E = -\Delta E_{\text{emit}}$ (negative sign is for *loosing/emitting* energy):

$$
\begin{aligned}
-\Delta E_{\text{emit}} &= cm\Delta T \\
-5700 &= 4180 \times 0.1 \times \Delta T \\
\Delta T &= -14\ \text{K}
\end{aligned}
$$

Thus, the temperature of the tea after 15 minutes will be:

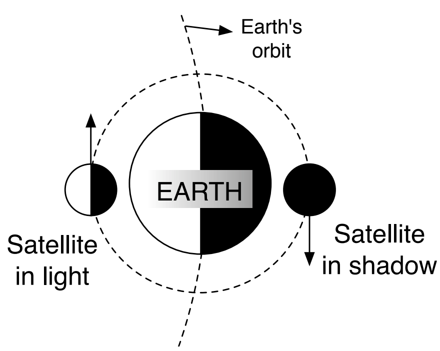
$$T_f = T_i + \Delta T = 373 - 14 = 359\ \text{K} = 86\ ^\circ\text{C}$$

This is still acceptable, but not so enjoyable for most people. It agrees with daily experiences.

Answer 3:

The average radiation absorbed by the satellite due to solar radiation near Earth:

$$
\begin{aligned}
P_{absorption} &= S \times A_{\text{cross section}} \times \frac{1}{2} \\
&= S\,(\pi r^2)\,\frac{1}{2}
\end{aligned}
$$



[2]Human physiology is very sensitive to the body temperature. Even a few ($\sim$ 5–6)°C drop is enough to knock someone down unconscious, and >13°C drop in body temperature means a certain death.

The 1/2 factor is due to the fact that the satellite sees light only about 1/2 of its orbital period: A period much smaller than 24 hrs indicates that it is a low-orbit satellite, thus it is in complete dark when it is above the dark side of the earth.

The emission is only due to blackbody radiation:

$$\begin{aligned} R &= \sigma T^4 \\ P_{emission}/A_{surface} &= \sigma T^4 \\ P_{emission} &= A_{surface}\, \sigma T^4 \\ P_{emission} &= (4\pi r^2)\, \sigma T^4 \end{aligned}$$

When it is in thermal equilibrium:

$$\begin{aligned} P_{emission} &= P_{absorption} \\ (4\pi r^2)\, \sigma T^4 &= S \cdot (\pi r^2) \cdot \frac{1}{2} \\ T &= \left[\frac{S}{8\,\sigma}\right]^{1/4} \\ T &= \left[\frac{1350}{8 \times 5.7 \times 10^{-8}}\right]^{1/4} \\ T &= 233 \text{ K} = -40\ ^\circ\text{C} \end{aligned}$$

The space is "cold"! Note that the lower limits of temperature of normal operation for most electronics components are rated to be around this level. Luckily, when electronics are in operation, they emit heat, thus they heat themselves, and shift the equilibrium point to a more comfortable level for the system.

Answer 4:

Let us assume that the radiation is constant within the given narrow range of wavelengths. We calculate the power per area per wavelength at the center of the given range of wavelengths; at $\lambda = 500$ nm. We use Planck's formula (T=6000 K):

$$\left.\frac{\mathrm{d}R}{\mathrm{d}\lambda}\right|_{\lambda=500\text{ nm}} = \left.\frac{2\pi hc^2}{\lambda^5(e^{hc/\lambda kT} - 1)}\right|_{\lambda=500\text{ nm}} \approx 10^5\ (\text{W}/m^2)/\text{nm}$$

Then, the energy emitted per cm^2 per second is

$$E = \left.\frac{\mathrm{d}R}{\mathrm{d}\lambda}\right|_{\lambda=500\text{ nm}} \times A \times \delta t \times \delta\lambda$$

where $A = 1\ \text{cm}^2$, $\delta t = 1$ sec, $\delta\lambda = (501 - 499) = 2$ nm.

$$E = 10^5 \times 10^{-4} \times 1 \times 2 = 20 \text{ J}$$

Answer 5:

We basically take our Sun and put it at a distance D. Then;

$$S' = S\frac{4\pi d^2}{4\pi D^2}$$

where $d = d_{sun-earth}$. Number of photons per second is:

$$\begin{aligned} N &\approx \frac{P \times 1\,\text{sec}}{\langle E \rangle} \\ &\approx \frac{S' \cdot A}{E_{\lambda=550\,\text{nm}}} \\ &\approx \frac{S' \cdot A\lambda}{hc} \\ &\approx S\left(\frac{d}{D}\right)^2 \frac{\pi r^2 \lambda}{hc} \\ D^2 &\approx \frac{S\, d^2\, \pi r^2\, \lambda}{hcN} \\ D &\approx 1.9 \times 10^{18} \text{ m} \approx 200 \text{ light-years} \end{aligned}$$

Thus, this star must be closer than 200 light-years.

It must be noted that we made an approximation for this solution: The radiation coming to the earth surface have a broader range than $400 < \lambda < 700$ nm, so the effective S' that includes only those photons is smaller. Thus, the upper limit for the distance we found must, in fact, be smaller. However, since we know that 500 nm is about the maxima of the $\mathrm{d}R/\mathrm{d}\lambda$ for $T = 6000$ K, and it decreases sharply as we get away from the maxima, *most* of the energy emitted are due to the photons in that limited range. Thus it is a good approximation.