Boğaziçi University
Department of Physics

Phys 442 — Spring 2011

Problem Set #2

Due on April 13th, 2011

Problem 1

Trapezoidal rule is given by

$$\int_{x_1}^{x_2} f(x)\,dx = h\left[\frac{1}{2}f_1 + \frac{1}{2}f_2\right] + \mathcal{O}(h^3 f'')$$

Show that

a) This formula yields exact result for a polynomial of degree-1, as expected due to being $f''=0$ which yields $\mathcal{O}(h^3 f'')$=0.

b) The error of this integral is indeed proportional to $h^3 f''$ for a polynomial of degree 2 by finding the difference between exact solution and Simpson's solution for $f(x) = a_0 + a_1 x + a_2 x^2$.

Problem 2

Consider the following function

$$f(x) = e^x \sin x$$

a) Evaluate

$$\int_2^3 f(x)\,dx$$

using Simpson's formula for $N = 5, 10, 50, 100, 1000$.

b) Compare errors and discuss the results. What is the relation between the error and N?

Notes:

a) Show your results in a table.

b) Error is defined as the difference between the evaluation and the actual value.

c) Note that you can find the actual value algebraically.

Problem 3

Consider the following integral:

$$I = \int_0^1 f(x)\,dx = \int_0^1 \frac{dx}{1+x^2} = \pi/4 = 0.78540$$

Write a Monte Carlo code that evaluates I and σ_I for $N = [10, 20, 50, 100, 200, 500, 1000, 2000, 5000]$ with $w(x) = 1$ and $w(x) = A\cos(kx)$. Find k, such that $\cos(kx) = f(x)$ at the lower and upper bound of the integral, find A by normalizing $w(x)$. Your code should give an output as follows:

N	I_1	σ_{I_1}	I_2	σ_{I_2}
10	...	...	...	...
20				

where I_1 is calculated with $w(x) = 1$, and I_2 is calculated with $w(x) = A\cos(kx)$.