**Boğaziçi University**
**Department of Physics**

Phys 496/68N Fall 2011

# Problem Set 4

**Due on November 18th, 2011**

## Problem 1

Consider the following equation:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -y$$

with $y(0) = 1$ as initial condition. We know the solution for this simple differential equation:

$$y = e^{-x}$$

Take the step size of $h = 0.1$. Evaluate $y(x)$ between $[0, 6]$ with the given step size.

## Problem 2

Consider the following equation:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -xy$$

with $y(0) = 1$ as initial condition. We know the solution for this simple differential equation:

$$y = e^{-x^2/2}$$

Test the accuracy of Euler's method using $y(1)$ and $y(3)$ as for the integration limits. Use the following step sizes for your test, and form a table showing your results:

$$h = [0.5, 0.1, 0.05, 0.01, 0.005, 0.001]$$

## Problem 3 – *Damped oscillation*

Consider a block of mass $m$ attached to a spring with spring constant $k$ which is submerged into a viscous liquid such that the drag force is linearly proportional to its velocity, $\vec{f}_{\text{drag}} = -b\,\vec{v}(t)$, as shown in the figure. The spring is initially stretched by an amount of $d$ from its natural equilibrium, such that $x_0 = d$ and $v_0 = 0$, then released at $t = 0$.

a) Form the related differential equation.

b) Take $m = 1$ kg, $b = 0.15$ Ns/m, $k = 1.2$ N/m, $d = 0.5$ m, and solve it numerically until the amplitude of the oscillation is roughly 5% of the initial amplitude. Make a plot of $x(t)$ vs $t$.

c) Find the solution analytically, and compare the numerical result with the analytical one on a plot.