Boğaziçi University
Department of Physics

Phys 496/68N Fall 2011

Problem Set 5

Due on November 25th, 2011

Problem 1

Write a code to fit the following data to a 3rd degree polynomial (the "model"). You will use your own matrix inversion code.

a) Find the polynomial coefficients,

b) Find the $\frac{\chi^2}{\text{d.o.f.}}$,

c) Can the model (3rd degree polynomial) explain the data? Answer your question based on $\frac{\chi^2}{\text{d.o.f.}}$ value.

d) Draw the model function with the best-fit parameters, and superimpose the data points with the associated error bars.

x	y	y_error
-4.00	-17.2532	1.8492
-3.75	-10.1902	1.3998
-3.50	-10.2282	1.9903
-3.25	-2.0565	2.1702
-3.00	0.7618	1.5184
-2.75	2.2905	2.5570
-2.50	3.5655	1.2069
-2.25	3.0319	1.8041
-2.00	4.9094	1.2577
-1.75	1.9622	1.8277
-1.50	1.5594	1.3380
-1.25	2.6641	1.7126
-1.00	2.7485	1.6296
-0.75	-2.0907	1.4075
-0.50	-3.6137	0.9676
-0.25	-3.7092	2.2825
0.00	-2.7829	3.0001
0.25	-4.0973	3.1117
0.50	-7.4012	1.7539
0.75	-9.4573	1.9770
1.00	-6.8871	1.7671
1.25	-7.3334	2.0380
1.50	-3.5037	1.5406
1.75	-4.7305	1.0403
2.00	0.2799	1.9818
2.25	6.0550	1.3874
2.50	9.1059	1.0488
2.75	14.1805	3.1871
3.00	22.2417	1.8833

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Problem 2

Fourth-order Runge–Kutta method can be implemented by the following equation:

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

where

$$\begin{aligned} k_1 &\equiv hf(x_n, y_n) \\ k_2 &\equiv hf(x_n + h/2, y_n + k_1/2) \\ k_3 &\equiv hf(x_n + h/2, y_n + k_2/2) \\ k_4 &\equiv hf(x_n + h, y_n + k_3) \end{aligned}$$

to solve a differential equation with the following notation:

$$y' = f(x, y)$$

Consider the Bernoulli equation:

$$\frac{dy}{dx} - \frac{2y}{x} = -x^2y^2$$

Solve this equation using Euler's Method <u>and</u> 4^{th}-order Runge-Kutta's Method between $[1, 5]$, and compare each of the solutions with the algebraic solution. Take $y(1) = 1$ as the initial condition.

The algebraic solution is

$$y = \frac{5x^2}{x^5 + C}$$

where C is a constant.