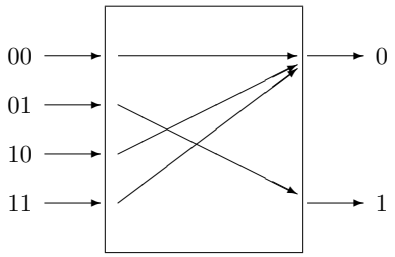
**Boğaziçi University**
**Department of Physics**

Phys 497 Spring 2008

# Midterm #2

**8 May 2008**

**Question 1:** *(10 pts)*
Consider a peculiar gate that detects whether one input is greater than the other. A properly working gate of this type has the model shown by the following figure:




a) Each of the four possible input states is equally likely. Calculate the two output probabilities $p(B_0)$ and $p(B_1)$, the input information $I_{in}$, the output information $I_{out}$, the noise $N$, loss $L$, and mutual information M. *(2 pts)*

b) Now, assume that this gate is faulty due to slight manufacturing error, so they behave as follows: *(2 pts)*

- the outputs for inputs (1 1) and (0 0) are correct.
- the output for input (0 1) will be wrong 10% of the time.
- the output for input (1 0) will be wrong 20% of the time.

Draw a box diagram which models the defective gate as a process. Include the transition probabilities in your diagram.

c) If the output is 1... *(2 pts)*

i) What is the probability that it was produced by the input (0 1)?
ii) What is the probability that it was produced by the input (1 0)?
iii) What is the probability that it was produced by the input (1 1)?

d) What are the input information $I_{in}$ and the output information $I_{out}$ (in the correct units)? *(2 pts)*

e) What are the noise $N$, the loss $L$, and the mutual information $M$? Is this process noisy, lossy, or both? *(2 pts)*

**Question 2:** *(10 pts)*
Consider a mass transportation system with a regular ticket price of 15 YTL. They offer 2/3 discount for the students, and pre-school children ("kids") can ride for free. The yearly balance sheet shows that the average ticket price per person is 5 YTL. If $P_{regular}$, $P_{student}$, $P_{kid}$ represent the probability distribution for these type of the passengers:

a) Write down the constraints for $P_{regular}$, $P_{student}$, $P_{kid}$ as defined above. *(2 pts)*

b) Find the most likely probability distribution (maximize the entropy.) *(2 pts)*

c) Find the entropy for the above probability distribution. *(2 pts)*

d) Now we are told that, they had used a technique to maximize the number of regular passengers. We find out that, according to the constraints given above, $P_{regular} = 1/3$ is the maximum that we can have. *(4 pts)*

i) Do you expect to have a higher or lower entropy with this additional information?
ii) Find the new probability distribution.
iii) Find the new entropy with the above probability distribution.

e) (**Optional**) Show that the maximum value of $P_{regular} = 1/3$ using the constraints given above. *(2 pts)*